Mechanical Engineering Project Proposal: Steam-Turbine Thermal Power Plant Turbine-Gearbox-Generator

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1 Abstract

This design report outlines a proposed design and solution for a power transmission system for use in a new, smaller scale, Concentrated Solar Power (CSP) facility in Nevada.

As such, a gearbox speed reduction system is required for a 30 MW CSP plant commissioned by ACS, the owners of the Crescent Dunes Solar Energy Project. This design aims to connect a steam turbine rotating at 6000 RPM, to a generator operating at 450 RPM. There will be two parallel turbines, each handling 15 MW of power at most.

Key design constraints and objectives include a 25-year operational lifespan, daily operation cycles from 5 PM to 5 AM, hence variable power output throughout the day. The design of the gearbox must include input and output shafts, meshing gears, supporting bearings, couplings, and an appropriate gearbox housing, with specified internal and external locating features, as well as input and output ports for lubricant cycling. This report outlines the initial concept layout, load determinations, and preliminary design considerations. We will assume a gearbox efficiency of 96% to account for minor losses through the gearing.

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2 Introduction & Background

2.1 Introduction

This report outlines the initial design concept for a gearbox speed reduction system for use in the concentrated solar power (CSP) plant. Commissioned by ACS, the owners of the Crescent Dunes Solar Energy Project in Nevada, USA, we are tasked with designing the critical mechanical elements of a gearbox speed reduction system between the turbine and generator. [1]

This project supports sustainable energy initiatives by optimizing CSP technology, which generates electricity at night while photovoltaic systems are unavailable. As part of a large renewable energy plan, this design aims to improve CSP dependability and operating efficiency, therefore guaranteeing constant power supply throughout the day. Furthermore, due to this unique ability to operate independently of peak sunlight hours, this technology provides a renewable alternative for base-load energy, which traditionally depends on non-renewable resources. By optimizing the gearbox for efficiency, we directly contribute to the reliability and economic viability of CSP plants, which in turn aligns with the global need for cleaner, sustainable energy solutions.

This report will outline the initial concept design, load determinations, and preliminary design considerations for this gearbox.

2.2 Background

Concentrated Solar Power plants use an array of mirrors to focus sunlight onto a receiver tower. This heats a molten salt mixture (approximately 60% sodium nitrate, 40% potassium nitrate) to temperatures up to 580°C, which is then pumped down into a storage container that will contain its heat. This stored thermal energy is then used to generate steam, and hence electricity during nighttime hours (5 pm to 5 am). This kind of power plant works in tandem with photovoltaic plants (solar panels), which have much lower operating costs, but only work through daylight hours. [1]

The proposed CSP plant will employ two parallel turbine-gearbox-generator systems, each handling 15 MW of peak power. The proposed gearbox must efficiently reduce the turbine's high rotational speed of 6000 RPM to the generator's required 450 RPM. [1]

As such, we must consider the variable daily power output cycles and the intended 25-year operational lifespan of the gearbox in our analysis, whilst assuming a gear power transfer efficiency of at least 96%. We must design all shafts and gears, as well as the housing with two ports for lubricant circulation, and we must specify all bearings, collars, couplers, fixating features and their locations, and mounting bolts. Likewise, we must always consider the assembly process, and make sure that it is possible.

4 Literature Review

4.1 Review of Existing Similar Projects

4.1.1 Crescent Dunes Solar Energy Project

The Crescent Dunes Solar Energy Project, located in Nevada, USA, is a 110 MW CSP plant that uses molten salt technology for energy storage. While significantly larger than our proposed 30 MW plant, it provides a valuable basis to begin engineering the overall system design and gearbox requirements. [2]

Key features: [3]

- 10,347 heliostats (mirrors) focusing sunlight onto a central receiver
- 640-foot-tall solar power tower
- 1.1 million MWh annual energy production
- 10 hours of full-load energy storage
- Molten salt storage system with 288 °C (cold tank) and 566 °C (hot tank) temperatures
- Steam turbine operating at 565 °C and 165 bar

The project uses a steam turbine connected to a gearbox and generator system, like our proposed design. The gearbox is crucial for matching the high-speed turbine output to the lower speed required by the generator. Specific details about the gearbox are not publicly available, but we can assume that it's a multi-stage reduction system designed for high efficiency and reliability under cyclical stress conditions. [2]

Figure 4.1 Crescent Dunes Solar Energy Project [1]

4.1.2 Noor III CSP Tower Plant

Located in Ouarzazate, Morocco, the Noor III CSP Tower Plant is a 150 MW facility that also employs molten salt technology. Once again, significantly larger than our proposed plant. Like our proposed plant, Noor III uses a gear reduction system to reduce the speed between parallel steam turbines and generators. [4]

Key features: [5]

- 7,400 heliostats
- 250-meter solar tower
- 7.5 hours of thermal energy storage
- Steam turbine operating at 565 °C and 150 bar
- Annual production of 500 GWh

Figure 4.2 Noor III CSP Tower Plant [4]

4.1.3 Ivanpah Solar Electric Generating System (ISEGS)

The ISEGS in California is a 392 MW CSP plant using direct steam generation rather than molten salt storage. While its technology differs from our project, its gearbox design principles are still relevant. [6]

Key features: [6]

- Three separate tower systems (126 MW, 133 MW, and 133 MW)
- 173,500 heliostats
- No thermal storage (direct steam generation)
- Multiple steam turbine-generator units

Figure 4.3 Ivanpah Solar Electric Generating System [6]

4.2 Market Survey

The global CSP market is expected to grow significantly in the coming years, with a projected CAGR of 9.7% from 2020 to 2027, and is expected to grow at higher rates in the future. This is primarily due to the increasing demand for renewable energy sources, government monetary incentives for adopting renewable technologies, advancements in thermal energy storage technology, and a growing focus on reducing carbon emissions in the power sector. [7],[8]

Potential customers for CSP plants and associated gearbox systems include utility companies, independent power producers, government agencies, particularly those focused on renewable energy systems, international development companies, and other private and public electricity companies. [7]

Typical costs for CSP projects range from $3,000 to $11,000 (USD)per kW of installed capacity [9]. For a 30 MW plant, this would translate to a total project cost between $90 million and $330 million. The gearbox system typically accounts for a very small fraction of the cost, less than 1% of the total project cost, placing our gearbox design at a cost of up to 3.3 million.

4.3 Module Justification

Gearbox design depends critically on the choice of gear module since it directly affects the strength, size, and load-carrying ability of the gear. The module (m) is defined as expressed in millimeters per tooth the ratio of the pitch diameter (d) to the number of teeth (N).

$$m = d/N$$

Larger modules are usually required for high-power projects, such as our 15MW gearbox, to guarantee the gears can get through high pressures and transfer the necessary torque. Choosing a large module is fitting for gears needed to handle more loads and transmit more power [11]. Choosing a 25mm module seems appropriate, given the large torque and loads that will be present in a speed reduction system of this magnitude.

5 List of Design Objectives & Requirements

Table 5.1 List of Design Objectives and Requirements

Design Objective	Requirement/Specification	Justification/Explanation
Electrical Output	15 MW nominal rated electrical output per turbine	This ensures the system meets the power demands of the CSP plant. The gearbox must be designed such that the generator may output up to 15 MW in electrical power
Gearbox Efficiency	96% efficiency	Assumed efficiency. In line for typical gearbox transmission systems. Gearbox design must account for power losses for each gear mesh.
Generator Efficiency	96% efficiency	Assumed Generator efficiency. Gearbox design must account for power losses within the generator
Turbine Speed	6000 RPM	Input speed. The speed at which the turbine operates, gearbox design must handle this high speed
Synchronous Generator Speed	450 RPM	Output speed. The speed to which the input must be reduced. This is the operating speed of the generator
System Design Lifetime	25 years	Typical lifespan for CSP plants before significant maintenance is required. Gearbox will be designed for an infinite lifetime.
Load Variation	Consider variable loads based on daily energy cycles	Load variations due to fluctuating energy demands must be accounted for in fatigue calculations due to power output varying through night operation. This must be considered in the design
Gearbox Layout	Must include housing, shafts, gears, bearings, seals, and shaft couplers	The gearbox must integrate all mechanical elements necessary for power transmission. Gears and Shafts must be designed, couplings, collars, and seals must be selected.
Assembly & Maintenance	Easy assembly and maintenance within the gearbox housing	Ensures initial installation and maintenance are possible. Gearbox design must be able to be assembled.
Mounting & Bolting	Secure gearbox housing to the steel support frame using appropriate fasteners	Bolts will be used to secure the housing to the steel base
Shaft Couplings	Input/output shafts aligned at equal height above the base plane	Generator input and turbine output must be at equal heights. Gearbox design must be planar
Gear Train Design	Multi-stage geartrain with appropriate gear ratios	Multi-stage reduction minimizes gearbox size, achieving 6000 to 450 RPM speed reduction
Material & Safety Factor Selection	Select materials and safety factors appropriate to design. Minimum safety factor of 1.4	Materials should be chosen, and gearbox elements should be designed for infinite lifetime, with a minimum safety factor of 1.4

Bearing & coupler Selection	Select appropriate bearings and couplers for shafts	Appropriate bearings, with the desired load and speed capabilities should be chosen

6 Concept Design

6.1 Overall Concept and Layout

The gearbox must convert an input at 6000-rpm into a 450-rpm output. Our team has decided that the most appropriate way to do this is with 3 stages, hence 3 speed ratios. This will result in two intermediate shafts.

The total speed ratio is given by the expression below:

$$|e| = \frac{|\omega_o|}{|\omega_i|} = \frac{\left|\left(\frac{2\pi}{60}\right)(rpm_o)\right|}{\left|\left(\frac{2\pi}{60}\right)(rpm_i)\right|} = \frac{\left|\left(\frac{2\pi}{60}\right)(-450)\right|}{\left|\left(\frac{2\pi}{60}\right)(6000)\right|} = \frac{450}{6000} = \frac{3}{40} = 3:40$$

This speed ratio may be decomposed into multiple stages as follows. Note that this is equivalent required gear teeth and pitch radius ratios.

$$\frac{3}{40} = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{3}{10}\right)$$

Such that the gear rpm is reduced as shown:

$$6000\,rpm \rightarrow 3000\,rpm \rightarrow 1500\,rpm \rightarrow 450\,rpm$$

This allows us to perform this gear reduction in three stages. The first two with 1:2 gear ratios, and the third with a 3:10 ratio. Figures 7.1 and 7.2 below show a sketch of the gearbox layout.

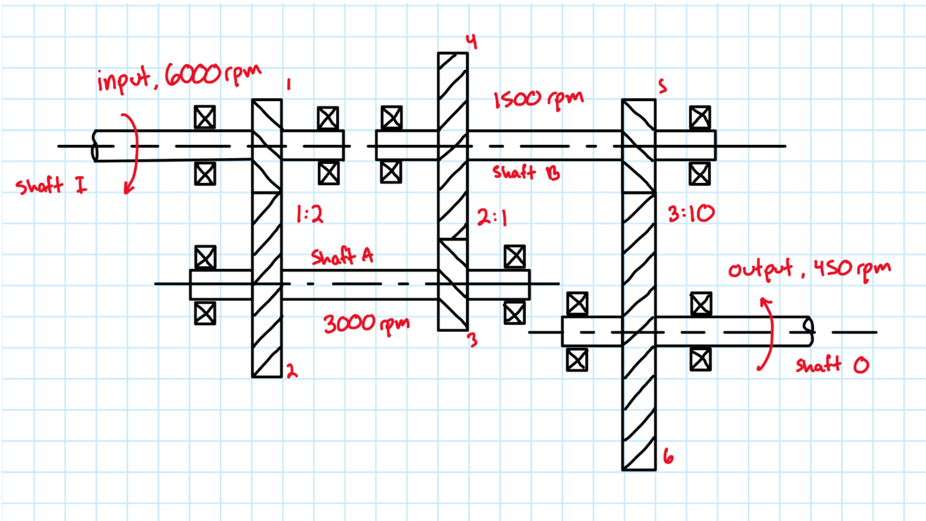


Figure 6.1 - top view of gearbox concept layout

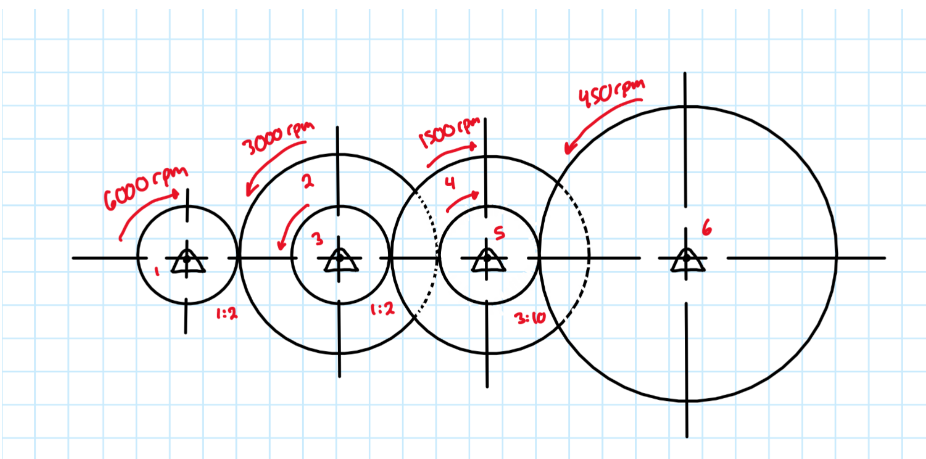


Figure 6.2 - spread out, side view of gearbox concept layout

6.2 Stage Calculations

This section outlines the speed, torque and power specification for each shaft. These calculations are for the maximum nominal power, achieved throughout the night.

Firstly, we are designing the gearbox such that the nominal electrical power output of the plant is 15 MW. The generator that will be used has an efficiency of 96% (0.96), which means the output power of the gearbox must account for energy losses in the generator. This means that the power at the output shaft is as follows:

$$P_{output} = \frac{P_{electrical}}{0.96} = \frac{15\ MW}{0.96} = 15.625\ MW$$

Likewise, the gearbox is assumed to have an overall efficiency of 96% (0.96), which means that the power at the input shaft is as follows:

$$P_{input} = \frac{P_{Output}}{0.96} = \frac{15.625\ MW}{0.96} = 16.276\ MW$$

Furthermore, we may assume that the gearbox efficiency losses are spread evenly throughout all shafts. This way, we find the efficiency loss between each shaft to be $\left(1 - \sqrt[3]{0.96}\right) \times 100 = 1.35\%$. Experimental data has found that spur gear mesh efficiency with gear ratios less than 6:1 is between 98%-99% [10], which aligns well with this assumption. In addition to gear mesh losses, there are also efficiency losses due to bearings, windage and churning. Overall, an efficiency loss of 1.35% per shaft remains relatively consistent with experimental data and thus will be used to find the power in shafts A and B. We will also be assuming that all power losses occur at the gear mesh.

$$\sqrt[3]{0.96} \cdot P_{gearbox,in} = P_{shaft,A}$$

$$P_{shaft,A} = 16.056MW$$

$$\sqrt[3]{0.96} \cdot P_{shaft,A} = P_{shaft,B}$$

$$P_{shaft,A} = 15.839MW$$

Since we now know the gearbox output power, our gear ratios, and both input and output rpm, we can quite easily determine the torque experienced in all shafts. Using the output as a reference calculation:

Output Shaft: $450\ rpm = 15\pi\ \frac{rad}{s},\ T_O = \frac{15.625MW}{15\pi\ \frac{rad}{s}} = 331.572\ kNm$

Table 6.1 Shaft power, speed, and torque specifications

Shaft	Power (MW)	Angular speed (rpm)	Torque (kNm)
Input (I)	16.276	6000	25.904
A	16.056	3000	51.108
B	15.839	1500	100.835
Output (O)	15.625	450	331.572

Table 7.1 above outlines the maximum power and torque experienced in all shafts throughout the 24-hour cycle. The maximum power output is maintained for 9 hours of each cycle, and then slowly declines to 1MW, where it remains for 8 hours, and then rises again to 15 MW. Thus, for a large portion of the cycle, the loads are continuously changing. Figure __ below shows how the torque in each shaft responds to the changing nominal output power.

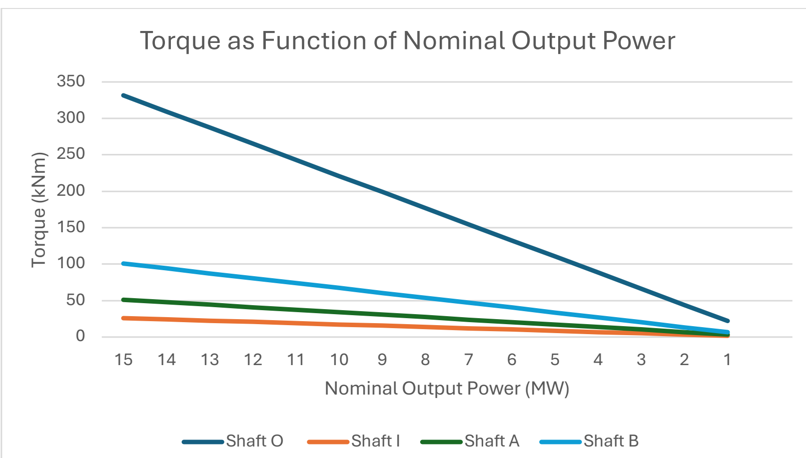


Figure 6.3 Torque as a function of nominal output power

6.3 Shaft Concept Design

6.3.1 Shape of the Shaft

All gears must be either machined directly onto the shaft, or fixated axially and radially onto the shaft, using either keys or splines, and collars or shoulders. This section aims to establish the general shape of each shaft, including shoulders for gears, shoulders for bearings, grooves for c-clips, threads for bearing locknuts, etcetera.

Figures 7.5 through to 7.8 show the conceptual shape for the shafts. Note that dimensions are not to scale, but coordinate axes are given for reference.

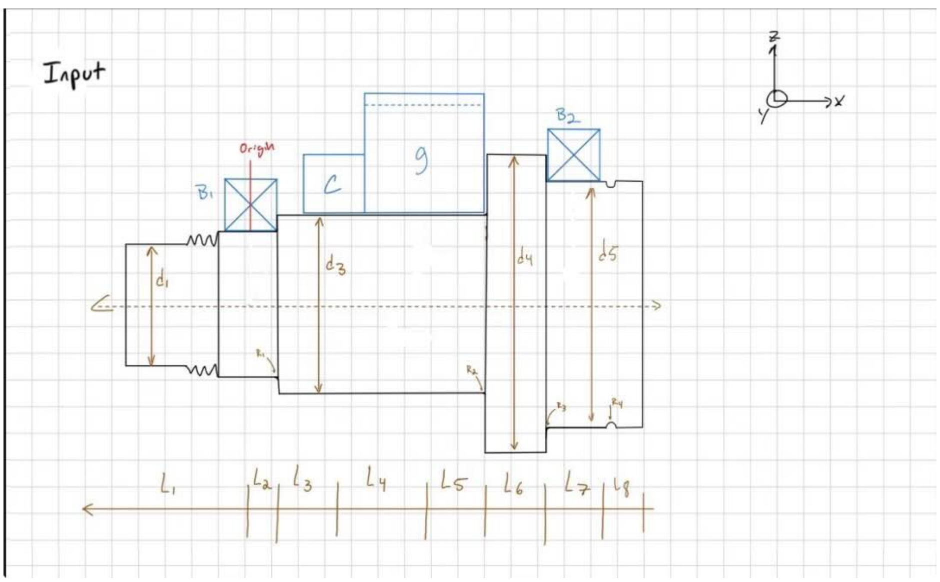


Figure 6.4 Shaft I concept shape

Shaft I has stress risers at both the bearings' shoulders, and at the gears' shoulder. Alongside those locations, stress analysis will be conducted at the leftmost side, where the keyway for the coupling is located, centers of each bearing, where reaction forces occur, and at the center of the gear.

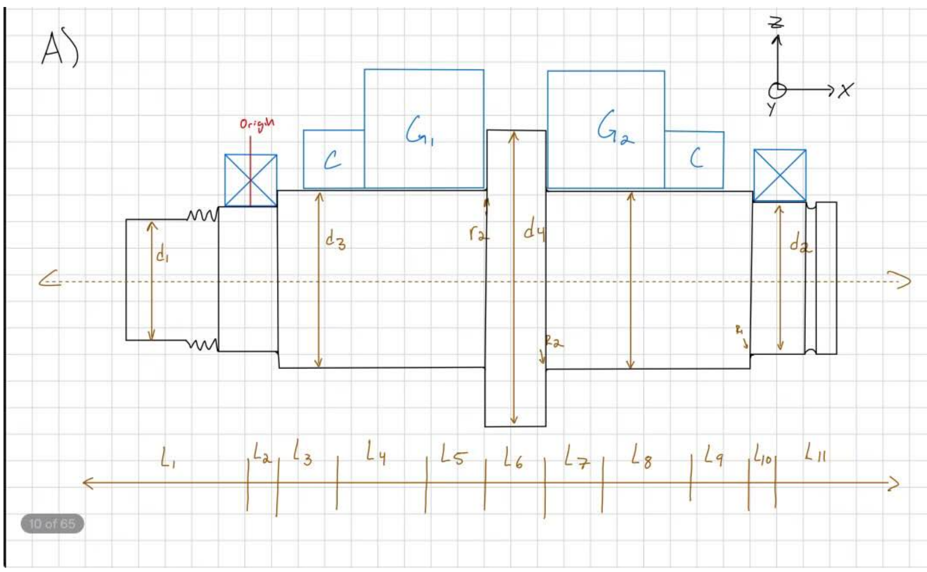


Figure 6.5 Shaft A concept shape.

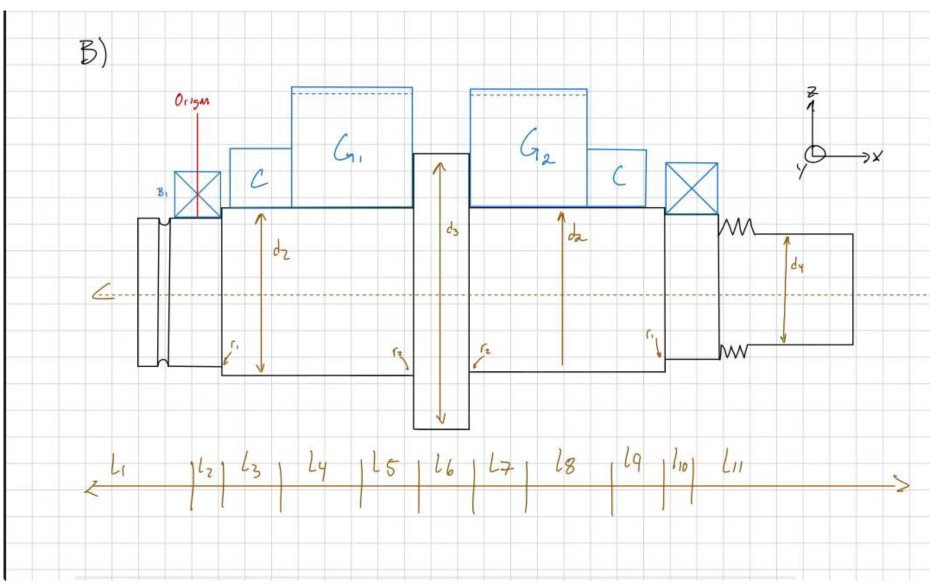


Figure 6.6 Shaft B concept shape.

Shaft A and B have stress risers at both the bearings' shoulders, and at both the gears' shoulders. Alongside those locations, stress analysis will be conducted at the centers of each bearing, where reaction forces occur, and at the centers of each gear.

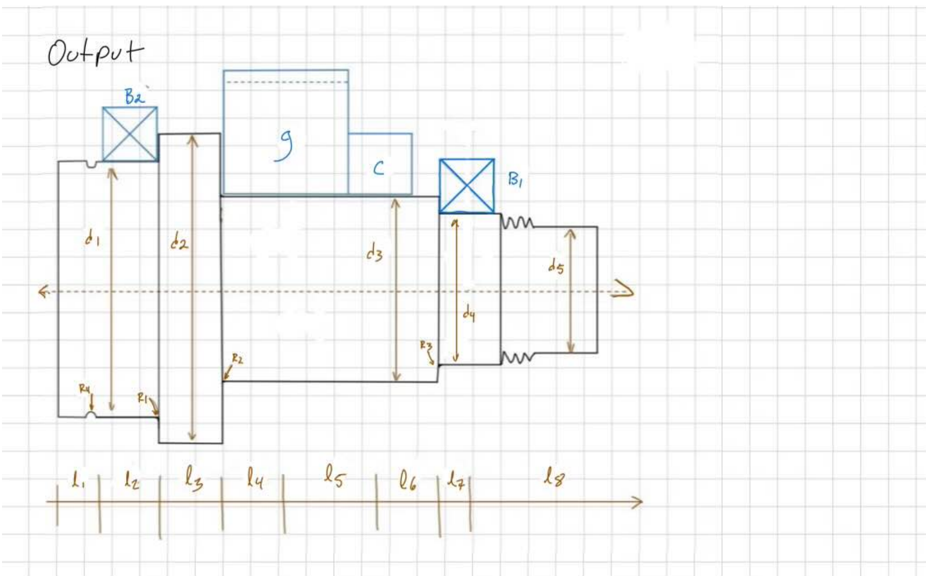


Figure 6.7 Shaft O concept shape.

Shaft O has stress risers at both the bearings' shoulders, and at the gears' shoulder. Alongside those locations, stress analysis will be conducted at the rightmost side, where the keyway for the coupling is located, centers of each bearing, where reaction forces occur, and at the center of the gear.

All of these features are better illustrated in the CAD drawings in a further section.

6.3.2 C-Clip justification

Since we are making use of spur gears, the axial forces will be very small, which will allow us to make use of c-clips to secure the floating bearings to the shafts. Whilst c-clips are a weak way to axially secure a component to a shaft, they are cheap and may be replaced without the need to take the entire assembly apart. This means that, if, for whatever reason, the gearbox is axially overloaded, the c-clips will fail first, protecting the other components from damage and a costly repair bill. Likewise, by only making use of c-clips to locate floating bearings, the gearbox will be able to operate smoothly even if the c-clips fail.

6.3.3 Fatigue Stress Analysis Locations

Every change in geometry along the length of the shaft will give rise to a stress concentration, all of which must be considered for the fatigue analysis. This section aims to establish what those locations are.

i. **Input and Output shafts, at coupling locations:** The keyway here introduces a stress concentration, which must be taken into consideration due to the high torque loads experienced at these locations.

ii. **Centre of each gear:** The keyway introduces a stress concentration. Furthermore, it is the location of assumed torque transfer, meaning the torque will suddenly change. The weight of the gear is also applied at its centre, when assuming it will be a point load. This assumption is made for the sake of simplicity. Radial and tangential forces are also assumed to be transferred, as point loads, to the centre of the gear. This means that the centre of one of the gears on each shaft will be the location of maximum bending moment and potentially shear force, hereby making it important to analyze the centre of each gear.

iii. **Centre of Bearings:** Reaction forces occur at the centre of each bearing, though no stress concentrations are present at these points. These may be locations of maximum shear force.

iv. **All Shoulders:** All gear and bearing shoulders introduce a stress concentration, which may be minimized by picking a large fillet radius. Likewise, these locations may experience high bending moments and shear forces and are hence important to analyze.

v. **Housing Mount Points:** The mounting points for the bearings must be able to hold the static load and will be placed under static compression. These points will still experience fatigue stress, due to the fluctuating power affecting the tangential, and radial reaction forces. The housing will be designed in more detail for stage 3, and the appropriate fatigue analysis will be conducted then.

vi. **Grooves for C-clips and locknut threads:** Whilst these locations contain stress risers, stress analysis is not necessary for these locations, as they are either under no load at all, or under constant torque in the case of some of the threads. Threads that are under constant torque are on the same sections of the shaft as the stress concentrations for the couplings' keyways, which are loaded under low cycle, fluctuating stress, and already have high safety factors.

6.4 Detailed Shaft Calculations

Assuming steady torsion and fully reversed bending; the analysis is conducted throughout the portion of the cycle with the maximum power.

Instead of calculating the shaft diameter using a given safety factor we will instead calculate the safety factor using a given diameter. This will be done such that less assumptions have to be made, is thus more accurate, and the calculation will be simpler and may be easily automated and iterated using code and a spreadsheet. We will be aiming for a safety factor of approximately 1.6 or greater, but no lesser than 1.4, for all sections of all shafts and gears.

These calculations will be done by finding the maximum principal stresses at the top/bottom, and centers of each shaft, at each critical location, as outlined in section 5.4. The greater of which will be used to find the safety factor.

Moreover, we must also consider the weight of each shaft, as well as the weight of the gears, as the shafts and gears will be rather heavy, and affect the overall loading conditions. The volume, and consequently the mass and weight, of all gears can be found using reference circle method [12], which will produce a relative error of no more than 2%. The masses of the shafts [13] and gears [14] may then be found using the respective materials' densities. For the sake of simplicity, all weights will be assumed to be point loads.

Throughout this analysis, the normal stress caused by the bending moment, shear stress due to torsion, and transverse shear stress due to shear forces must be considered for each critical location [1]. Note that transverse shear stress due to forces in the vertical direction will be zero at the top and bottom of the shaft, but maximum across the middle of the cross-section, so it will affect the analysis at the center of the shaft. Likewise, forces in the horizontal direction will generate transverse shear at the top/bottom, but not at the center of the shafts.

Further, all stress concentration factors for each critical location with a stress riser are given by interpolating from Figures 11.1 and 11.2. In the event of combined loading, the maximum stress concentration factor has been used [1].

6.4.1 Shaft I

The input shaft, shaft I, is loaded as such, taking the center of the leftmost bearing as the origin (x = 0 m)

Weight of gear: -2.776 kN k^, x = 0.216 m

Weight of Shaft: -0.820 kN k^, x = 0.145 m

Tangential Force: -103.62 kN k^, x = 0.216 m

Radial Force: 37.7 kN j^, x = 0.216 m

Torque: 25.9 kNm, up to x = 0.216 m

Reaction forces are calculated such that: (< i^, j^, k^ >)

Bearing 1 Reaction = <0, -6.83, 47.98> kN, at x = 0 m

Bearing 2 Reaction = <0, -20.89, 59.23> kN, at x = 0.39 m

This may be further visualized in figure 7.8

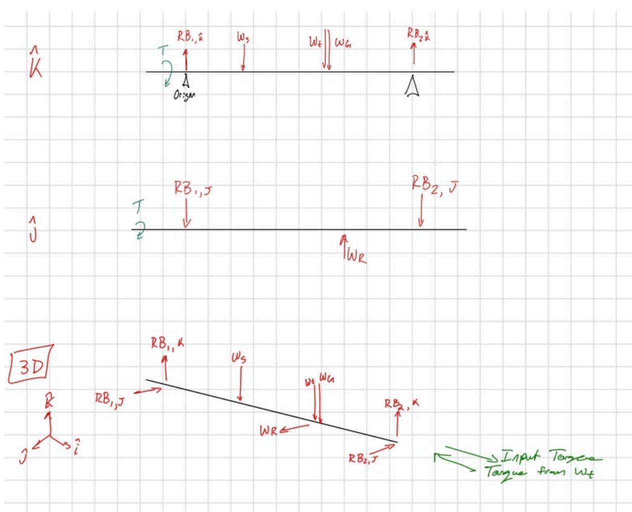


Figure 6.8 Shaft I free body diagrams.

This state of loading can be further explored through the shear force and bending moment diagrams found in figures 7.9 through to 7.12 below (Detailed Calculations for figures 7.9 to 7.12 can be found in Appendix B):

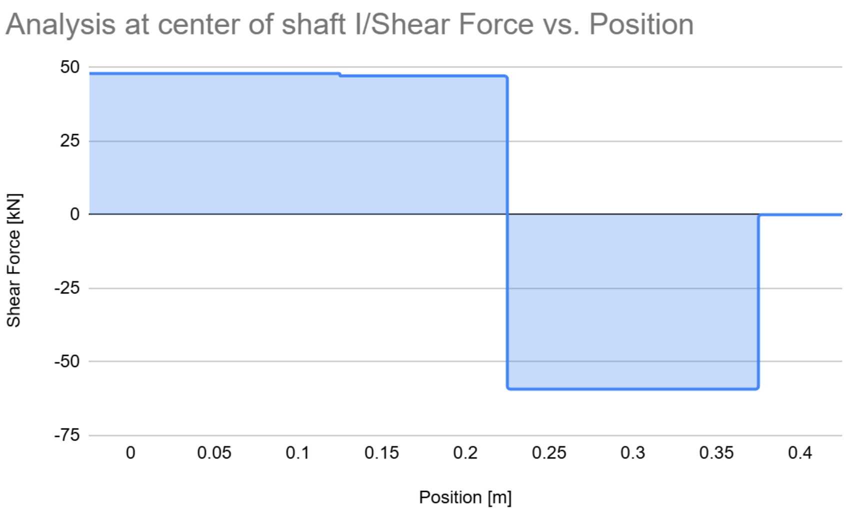


Figure 6.9 Shaft I Shear force diagram for Vertical forces, used in analysis at center of shaft

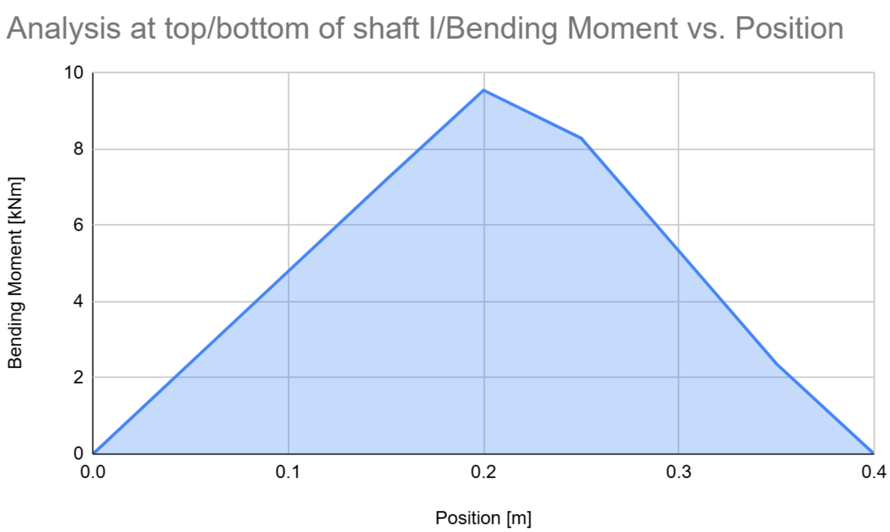


Figure 6.10 Shaft I Bending Moment diagram for Vertical forces, used in analysis at top/bottom of shaft

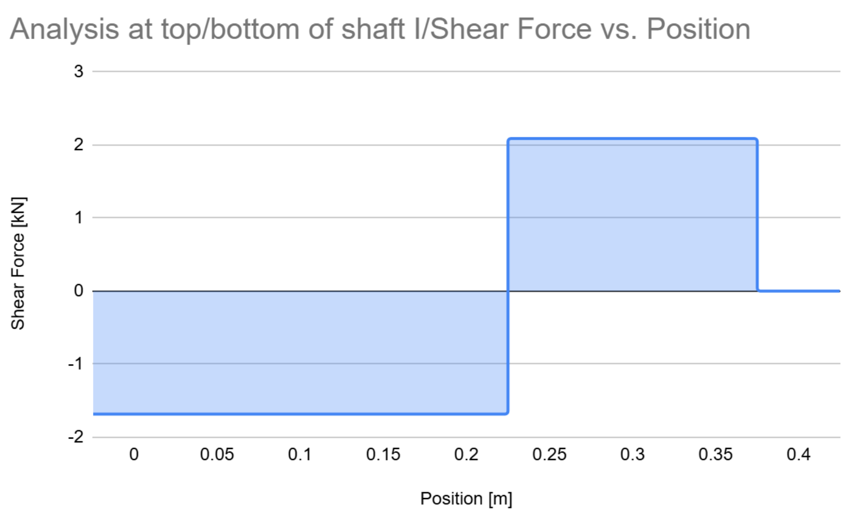


Figure 6.11 Shaft I Shear force diagram for Horizontal forces, used in analysis at top/bottom of shaft

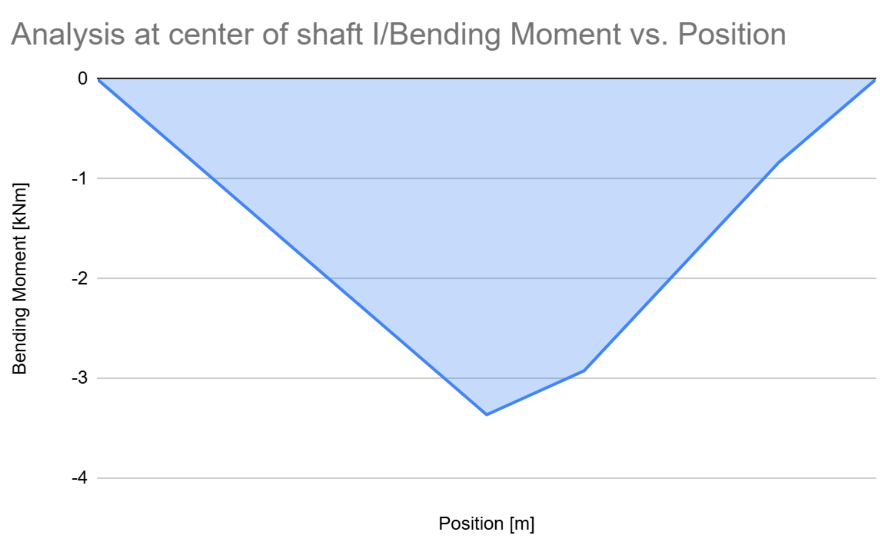


Figure 6.12 Shaft I Bending Moment diagram for Horizontal forces, used in analysis at center of shaft

These diagrams allow us to determine the shear force and bending moment at any point, given an x-position, which can be easily determined given the lengths of each shaft.

Subsequently, we can find the Bending Normal and Shear stresses for the top and center of each shaft's cross-section. Note that transverse shear from forces in the k^ direction will only have an effect at the center of the cross-section, and those from forces in the j^ direction will only have an effect at the top of the cross-section.

We may obtain the stress state through the following calculations:

$$\tau_{transverse,\ at\ top} = \frac{3V_j}{4A} = \frac{3V_j}{4\pi r^2}$$

$$\tau_{transverse,\ at\ sides} = \frac{3V_k}{4A} = \frac{3V_k}{4\pi r^2}$$

$$\sigma_{at\ top} = \frac{M_j r}{I} = \frac{M_j r}{\frac{1}{4}\pi r^4} = \frac{4M_j}{\pi r^3}$$

$$\sigma_{at\ sides} = \frac{M_k r}{I} = \frac{M_k r}{\frac{1}{4}\pi r^4} = \frac{4M_k}{\pi r^3}$$

$$\tau_{torsion} = \frac{Tp}{J} = \frac{2T}{\pi r^3}$$

$$\tau_{top,\ total} = |\tau_{transverse,\ at\ top|} + |\tau_{torsion}|$$

$$\sigma_{principle,nom} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{total}{}^2}$$

$$\sigma_y = 0$$

Given this information, we may now calculate the stress states at the top and sides of the shaft at each critical location. Tables 7.2 and 7.3 show this.

Table 6.2 Shear forces, Bending Moments, and stress states at top of shaft I

Critical Location	V j^ [kN]	M -j^ [kNm]	T i^ [kNm]	Normal Stress [MPa]	Shear Stress [MPa]
Coupler Keyway	0.00	0.00	25.90	0.00	60.05
Centre of Left Bearing	-16.83	0.00	25.90	0.00	48.10
First Bearing Shoulder	-16.83	3.89	25.90	-14.43	48.10
Centre of Gear	20.89	10.31	25.90	-25.63	32.23
Gear Shoulder	20.89	4.38	0.00	-10.90	32.23
Second Bearing Shoulder	20.89	1.42	0.00	-10.88	0.02
Centre of Right Bearing	20.89	0.00	0.00	0.00	0.02

Table 6.3 Shear forces, Bending Moments, and stress states at sides of shaft I

Critical Location	V k^ [kN]	M k^ [kNm]	Normal Stress [MPa]	Shear Stress [MPa]
Coupler Keyway	0.00	0.00	0.00	60.05
Centre of Left Bearing	47.98	0.00	0.00	52.23
First Bearing Shoulder	47.98	-1.36	5.06	52.23
Centre of Gear	-59.23	-3.63	9.04	36.14
Gear Shoulder	-59.23	-1.55	3.84	36.14
Second Bearing Shoulder	-59.23	-0.50	3.84	8.31
Centre of Right Bearing	-59.23	0.00	0.00	8.31

Furthermore, we may use the information from the previous tables to determine the safety factor at each point.

First, we find our stress concentration factors:

$$K_t = A\left(\frac{r}{d}\right)^b \quad , \quad given\ value\ of\ \frac{D}{d}, A\ and\ b\ can\ be\ found\ in\ Appendix\ A$$

$$K_f = 1 + q(k_t - 1) \quad , \quad given\ notch\ radius, q\ can\ be\ found\ in\ Appendix\ A$$

And we may determine the maximum nominal principal stress by taking the maximum of the absolute value of the principal stresses on top and sides.

Ultimately, we find that the maximum stress experienced by the shaft at each critical location is given by the following equation:

$$\sigma_{max} = K_f \sigma_{max,nom}$$

Lastly, the material endurance limit, S_e, and safety factor, are calculated as follows:

Material: Type 301 SS – cold rolled

$$S_{UT} = 1379MPa$$

$$S_e = C_{LOAD}C_{SIZE}C_{SURF}C_{TEMP}C_{RELI}S_{e'}$$

$$S_{e'} \approx 0.5S_{UT} \approx 689.5MPa$$

$$C_{LOAD} = 1.0$$

$C_{SIZE} = 1.189d^{-0.097}$ (d in mm) (changes depending on location)

$$C_{SURF} \cong A(S_{UT})^b \cong 4.51(1379)^{-0.265} = 0.664$$

$$C_{TEMP} = 1.0$$

$C_{RELI} = 0.702$ (99.99%)

And finally, the safety factor, N:

$$N = \frac{S_e}{\sigma_{max}}$$

We may use this information to calculate the parameters shown in tables 7.4 and 7.5 below.

Table 6.4 Stress Concentrations for Shaft I

Critical Location	Maximum Nominal Principal Stress [MPa]	Kt	q	Kf	Maximum Principal Stress [MPa]
Coupler Keyway	104.01	2.650	0.954	2.574	126.10
Centre of Left Bearing	90.47	1.000	0.000	1.000	90.47
First Bearing Shoulder	90.61	1.849	0.969	1.823	165.17
Centre of Gear	63.58	2.670	0.954	2.593	164.85
Gear Shoulder	62.71	2.335	0.954	2.273	142.53
Second Bearing Shoulder	14.90	2.407	0.954	2.342	34.89
Centre of Right Bearing	14.39	1.000	0.000	1.000	14.39

Table 6.5 Material Limits and Safety Factors for Shaft I

Critical Location	diameter at point [m]	Maximum Principal Stress [MPa]	Material limit at location [MPa]	Safety Factor
Coupler Keyway	0.13	126.10	238.33	1.89
Centre of Left Bearing	0.14	90.47	236.63	2.62
First Bearing Shoulder	0.14	165.17	236.63	1.43
Centre of Gear	0.16	164.85	233.58	1.42
Gear Shoulder	0.16	142.53	233.58	1.64
Second Bearing Shoulder	0.11	34.89	242.23	6.94
Centre of Right Bearing	0.11	14.39	242.23	16.83

6.4.2 Shaft A

In similar fashion to shaft I, we find that shaft A is loaded as such, taking the center of the leftmost bearing as the origin (x = 0 m)

Weight of gear 1: -11.82 kN k^, x = 0.17m

Weight of gear 2: -5.09 kN k^, x = 0.77 m

Weight of Shaft: -3.19 kN k^, x = 0.522 m

Tangential Force 1: 103.62 kN k^, x = 0.17 m

Tangential Force 2: -204.43 kN k^, x = 0.77 m

Radial Force 1: -37.71 kN j^, x = 0.17 m

Radial Force 2: -74.42 kN j^, x = 0.77 m

Torque: -51.1 kNm, from x = 0.17, up to x = 0.77 m

Reaction forces are calculated such that: (< i^, j^, k^ >)

Bearing 1 Reaction = <0, 52.02, -18.13> kN, at x = 0 m

Bearing 2 Reaction = <0, 60.10, 139.05> kN, at x = 1.06 m

This may be further visualized in figure 7.13

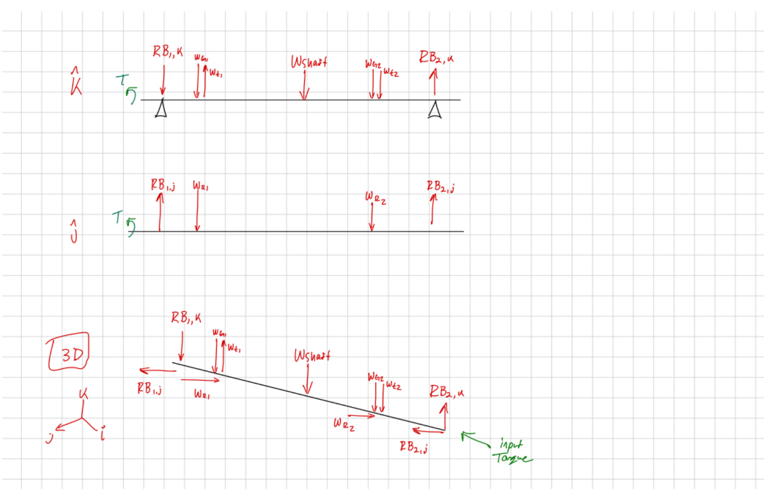


Figure 6.13 Shaft A free body diagram

Subsequently, we can develop the following shear force and bending moment diagrams (Detailed calculations for figures 7.14 to 7.17 can be found in Appendix B):

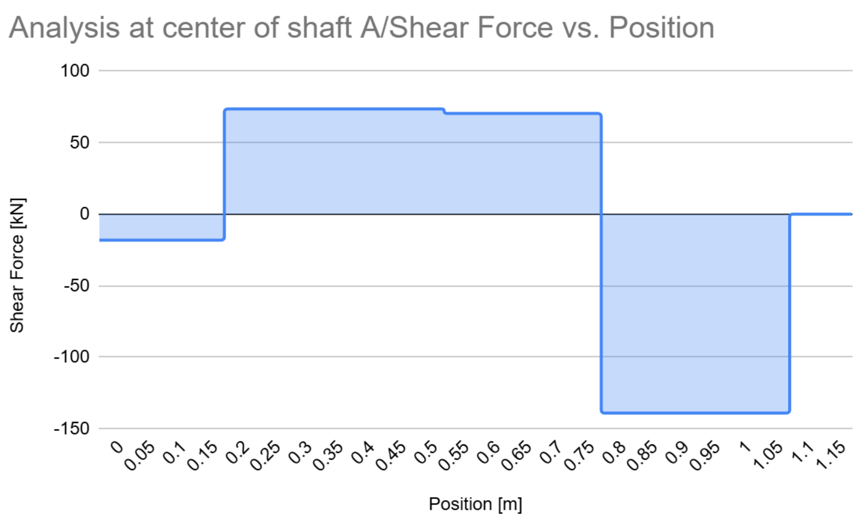


Figure 6.14 Shaft A Shear force diagram for Vertical forces, used in analysis at center of shaft

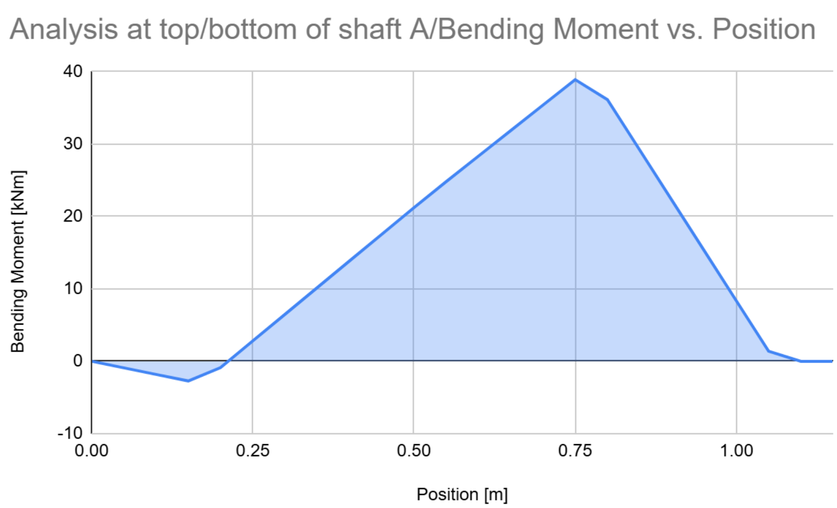


Figure 6.15 Shaft A Bending Moment diagram for Vertical forces, used in analysis at top/bottom of shaft

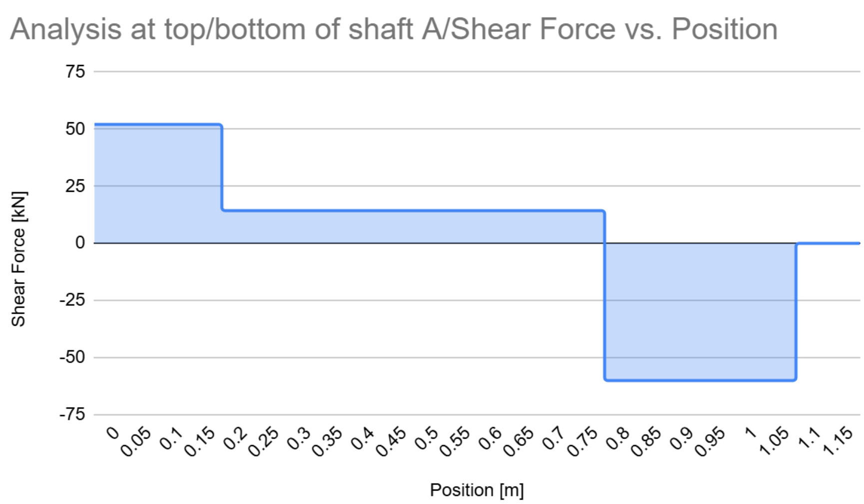


Figure 6.16 Shaft A Shear force diagram for Horizontal forces, used in analysis at top/bottom of shaft

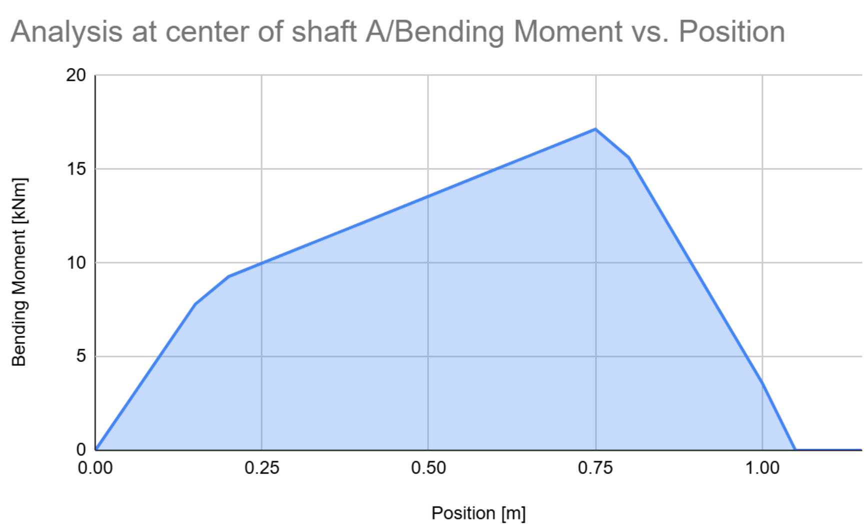


Figure 6.17 Shaft A Bending Moment diagram for Horizontal forces, used in analysis at center of shaft

And we can develop the following tables to determine the safety factor for each critical location:

Table 6.6 Shear forces, Bending Moments, and stress states at top of shaft A

Critical Location	V j^ [kN]	M -j^ [kNm]	T i^ [kNm]	Normal Stress [MPa]	Shear Stress [MPa]
Bearing 1 Centre	52.02	0.00	0.00	0.00	10.90
Bearing 1 Shoulder	52.02	-0.63	0.00	8.87	10.90
Gear 1 Centre	14.31	-3.08	-51.11	3.39	28.66
Gear 1 Shoulder	14.31	4.28	-51.11	-4.71	28.66
Gear 2 Shoulder	14.31	26.23	-51.11	-28.85	28.66
Gear 2 Centre	-60.10	40.32	-51.11	-44.35	30.42
Bearing 2 Shoulder	-60.10	7.65	0.00	-106.86	12.60
Bearing 2 Centre	-60.10	0.00	0.00	0.00	12.60

Table 6.7 Shear forces, Bending Moments, and stress states at sides of shaft A

Critical Location	V k^ [kN]	M k^ [kNm]	Normal Stress [MPa]	Shear Stress [MPa]
Bearing 1 Centre	-18.13	0.00	0.00	3.80
Bearing 1 Shoulder	-18.13	1.82	-25.44	3.80
Gear 1 Centre	73.66	8.84	-9.73	30.94
Gear 1 Shoulder	73.66	10.27	-11.30	30.94
Gear 2 Shoulder	70.48	14.57	-16.02	30.82
Gear 2 Centre	-139.05	17.43	-19.17	33.46
Bearing 2 Shoulder	-139.05	3.31	-46.19	29.14
Bearing 2 Centre	-139.05	0.00	0.00	29.14

Table 6.8 Stress Concentrations for Shaft A

Critical Location	Maximum Nominal Principal Stress [MPa]	Kt	q	Kf	Maximum Principal Stress [MPa]
Bearing 1 Centre	18.88	1.000	0.000	1.000	18.88
Bearing 1 Shoulder	26.28	2.344	0.954	2.282	59.96
Gear 1 Centre	54.47	2.750	0.954	2.669	145.38
Gear 1 Shoulder	54.77	2.409	0.954	2.344	128.38
Gear 2 Shoulder	57.41	2.409	0.954	2.344	134.57
Gear 2 Centre	68.87	2.500	0.954	2.431	167.40
Bearing 2 Shoulder	109.06	2.111	0.961	2.067	164.48
Bearing 2 Centre	50.48	1.000	0.000	1.000	50.48

Table 6.9 Material Limits and Safety Factors for Shaft A

Critical Location	diameter at point [m]	Maximum Principal Stress [MPa]	Material limit at location [MPa]	Safety Factor
Bearing 1 Centre	0.09	18.88	246.99	13.08
Bearing 1 Shoulder	0.09	59.96	246.99	4.12
Gear 1 Centre	0.21	145.38	227.50	1.56
Gear 1 Shoulder	0.21	128.38	227.50	1.77
Gear 2 Shoulder	0.21	134.57	227.50	1.69
Gear 2 Centre	0.21	167.40	227.50	1.36
Bearing 2 Shoulder	0.09	164.48	246.99	1.50
Bearing 2 Centre	0.09	50.48	246.99	4.89

6.4.3 Shaft B

In similar fashion to shaft I and A, we find that shaft B is loaded as such, taking the center of the leftmost bearing as the origin (x = 0 m)

Weight of gear 1: -23.14 kN k^, x = 0.296 m

Weight of gear 2: -6.06 kN k^, x = 1.196 m

Weight of Shaft: -6.7 kN k^, x = 0.824 m

Tangential Force 1: 204.43 kN k^, x = 0.296 m

Tangential Force 2: -448.15 kN k^, x = 1.196 m

Radial Force 1: 74.4 kN j^, x = 0.296 m

Radial Force 2: 163.1 kN j^, x = 1.196 m

Torque: 100.8 kNm, from x = 0.296, up to x = 1.196 m

Reaction forces are calculated such that: (< i^, j^, k^ >)

Bearing 1 Reaction = <0, -108.9, -13.6> kN, at x = 0 m

Bearing 2 Reaction = <0, -128.6, 293.3> kN, at x = 1.688 m

This may be further visualized in figure 7.18

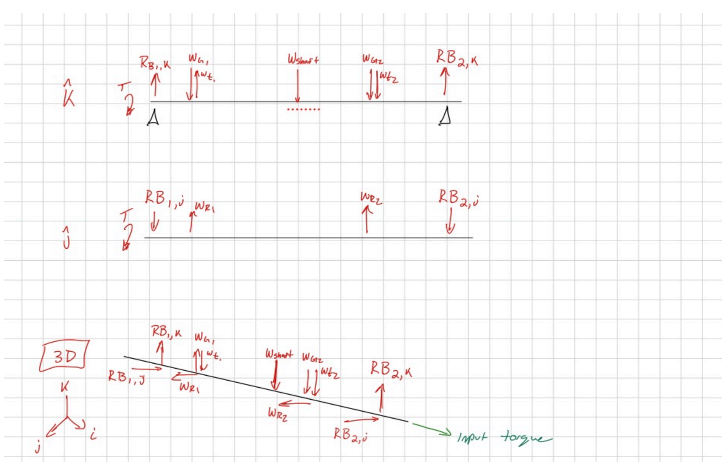


Figure 6.18 Shaft B free body diagram

Subsequently, we can develop the following shear force and bending moment diagrams (Detailed calculations for figures 7.19 to 7.22 can be found in Appendix B):

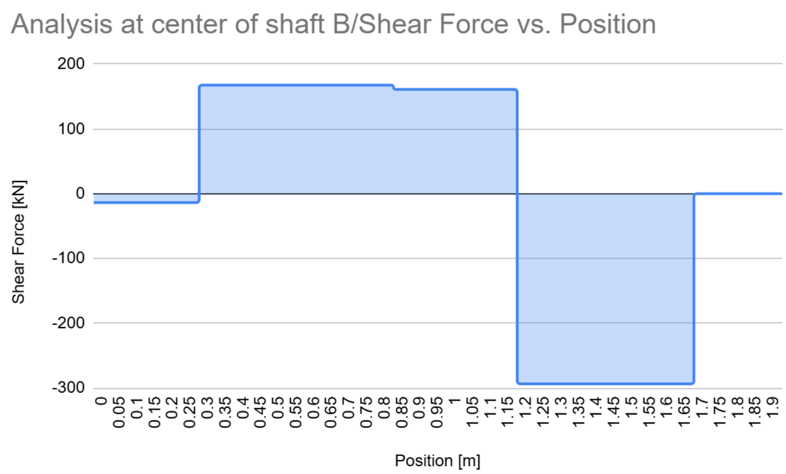


Figure 6.19 Shaft B Shear force diagram for Vertical forces, used in analysis at center of shaft

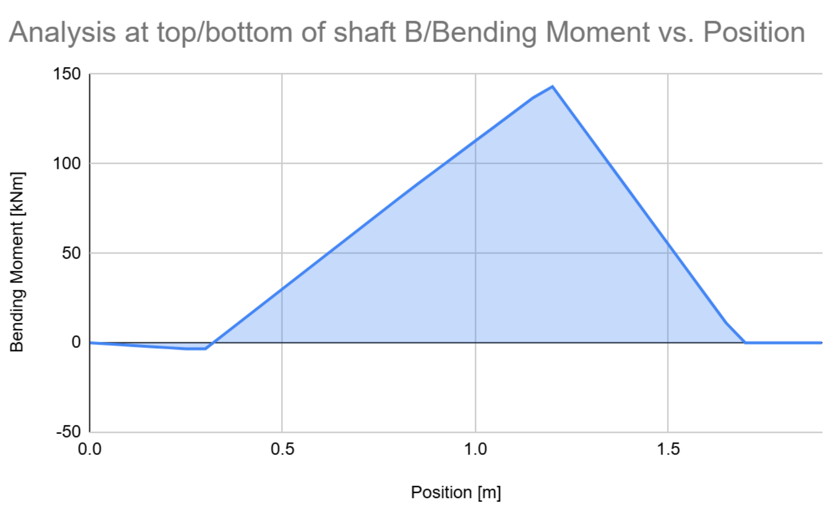


Figure 6.20 Shaft B Bending Moment diagram for Vertical forces, used in analysis at top/bottom of shaft

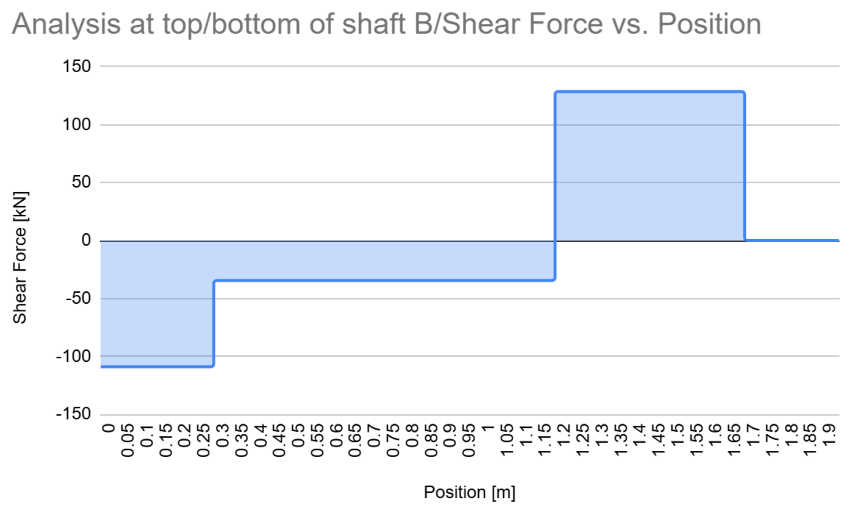


Figure 6.21 Shaft B Shear force diagram for Horizontal forces, used in analysis at top/bottom of shaft

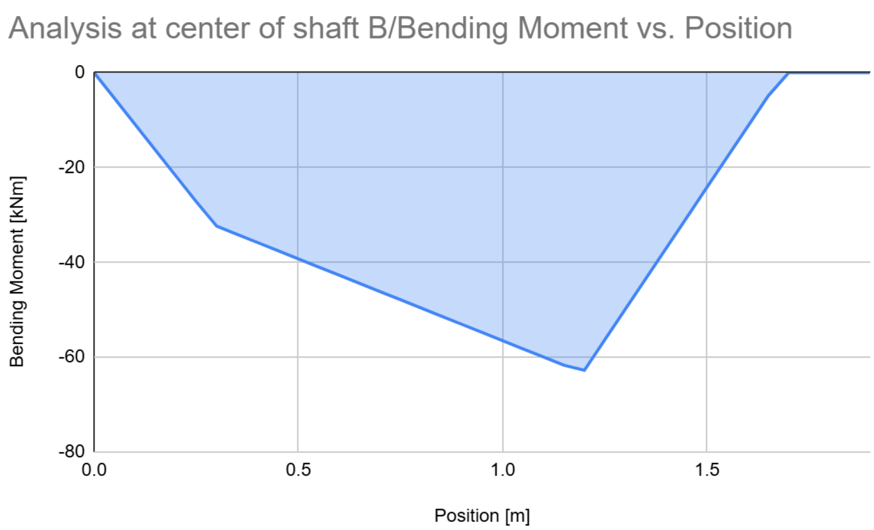


Figure 6.22 Shaft B Bending Moment diagram for Horizontal forces, used in analysis at center of shaft

And we can develop the following tables to determine the safety factor for each critical location:

Table 6.10 Shear forces, Bending Moments, and stress states at top of shaft B

Critical Location	V j^ [kN]	M -j^ [kNm]	T i^ [kNm]	Normal Stress [MPa]	Shear Stress [MPa]
Bearing 1 Centre	-108.90	0.00	0.00	0.00	12.84
Bearing 1 Shoulder	-108.90	-0.83	0.00	4.90	12.84
Gear 1 Centre	-34.50	-4.03	100.83	2.63	19.96
Gear 1 Shoulder	-34.50	29.49	100.83	-19.23	19.96
Gear 2 Shoulder	-34.50	88.01	100.83	-57.38	19.96
Gear 2 Centre	128.62	144.32	100.83	-94.08	22.51
Bearing 2 Shoulder	128.62	31.39	0.00	-185.01	15.16
Bearing 2 Centre	128.62	0.00	0.00	0.00	15.16

Table 6.11 Shear forces, Bending Moments, and stress states at sides of shaft B

Critical Location	V k^ [kN]	M k^ [kNm]	Normal Stress [MPa]	Shear Stress [MPa]
Bearing 1 Centre	-13.62	0.00	0.00	1.61
Bearing 1 Shoulder	-13.62	-6.64	39.16	1.61
Gear 1 Centre	167.62	-32.24	21.01	23.57
Gear 1 Shoulder	167.62	-39.13	25.51	23.57
Gear 2 Shoulder	160.88	-51.21	33.38	23.39
Gear 2 Centre	-293.33	-63.28	41.25	26.99
Bearing 2 Shoulder	-293.33	-13.76	81.12	34.58
Bearing 2 Centre	-293.33	0.00	0.00	34.58

Table 6.12 Stress Concentrations for Shaft B

Critical Location	Maximum Nominal Principal Stress [MPa]	Kt	q	Kf	Maximum Principal Stress [MPa]
Bearing 1 Centre	22.24	1.00	0.00	1.00	22.24
Bearing 1 Shoulder	39.26	2.58	0.95	2.50	98.31
Gear 1 Centre	45.92	2.75	0.95	2.67	122.56
Gear 1 Shoulder	48.15	2.46	0.95	2.39	115.28
Gear 2 Shoulder	66.98	2.46	0.95	2.39	160.38
Gear 2 Centre	101.84	2.65	0.95	2.57	154.11
Bearing 2 Shoulder	186.87	1.88	0.95	1.84	195.91
Bearing 2 Centre	59.90	1.00	0.00	1.00	59.90

Table 6.13 Material Limits and Safety Factors for Shaft B

Critical Location	diameter at point [m]	Maximum Principal Stress [MPa]	Material limit at location [MPa]	Safety Factor
Bearing 1 Centre	0.12	22.24	240.20	10.80
Bearing 1 Shoulder	0.12	98.31	240.20	2.44
Gear 1 Centre	0.25	122.56	223.69	1.83
Gear 1 Shoulder	0.25	115.28	223.69	1.94
Gear 2 Shoulder	0.25	160.38	223.69	1.49
Gear 2 Centre	0.25	154.11	223.69	1.45
Bearing 2 Shoulder	0.12	195.91	240.20	1.47
Bearing 2 Centre	0.12	59.90	240.20	4.01

6.4.4 Shaft O

The output shaft, shaft O, is loaded as such, taking the center of the leftmost bearing as the origin (x = 0 m)

Weight of gear: -89.77 kN k^, x = 0.45 m

Weight of Shaft: -10.8 kN k^, x = 0.472 m

Tangential Force: 448.15 kN k^, x = 0.45 m

Radial Force: -163.11 kN j^, x = 0.45 m

Torque: -331.6 kNm, from x = 0.45

Reaction forces are calculated such that: (< i^, j^, k^ >)

Bearing 1 Reaction = <0, 80.175, -171.10> kN, at x = 0 m

Bearing 2 Reaction = <0, 82.94, -176.45> kN, at x = 0.885 m

This may be further visualized in figure 7.23

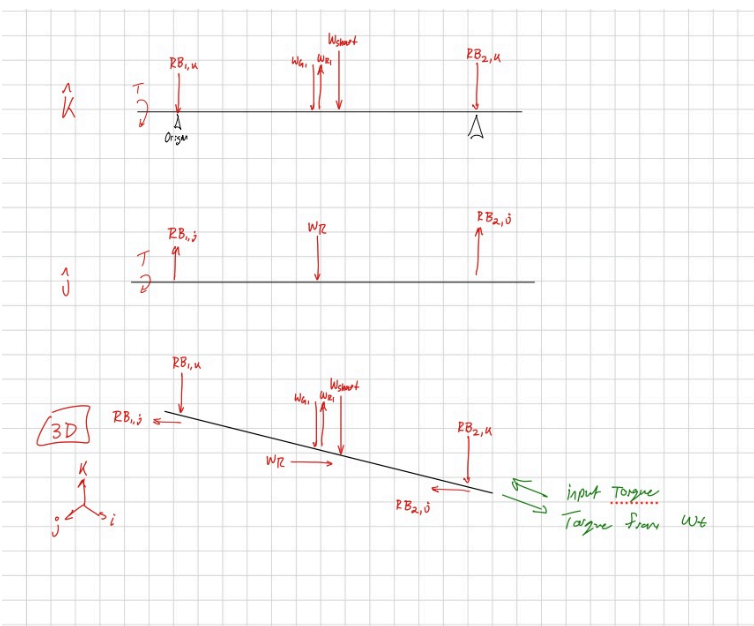


Figure 6.23 Shaft O free body diagram

Subsequently, we can develop the following shear force and bending moment diagrams (Detailed calculations for figures 7.24 to 7.27 can be found in Appendix B):

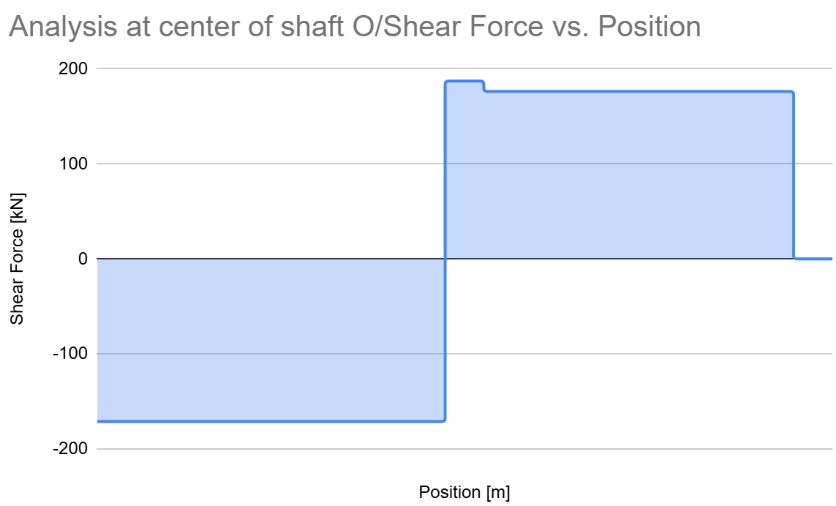


Figure 6.24 Shaft O Shear force diagram for Vertical forces, used in analysis at center of shaft

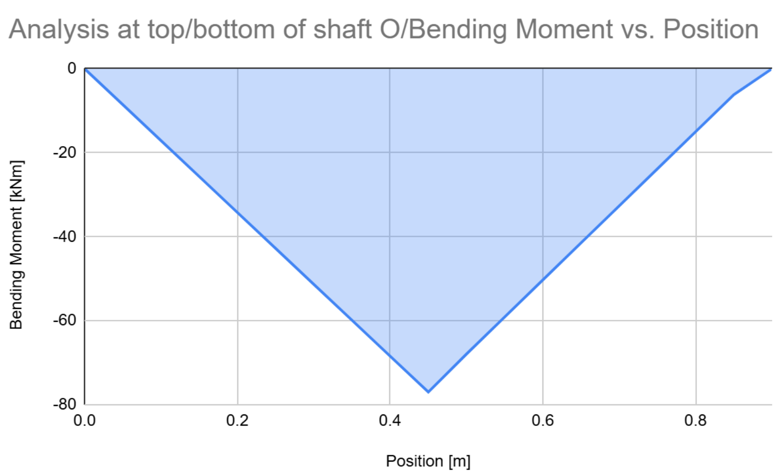


Figure 6.25 Shaft O Bending Moment diagram for Vertical forces, used in analysis at top/bottom of shaft

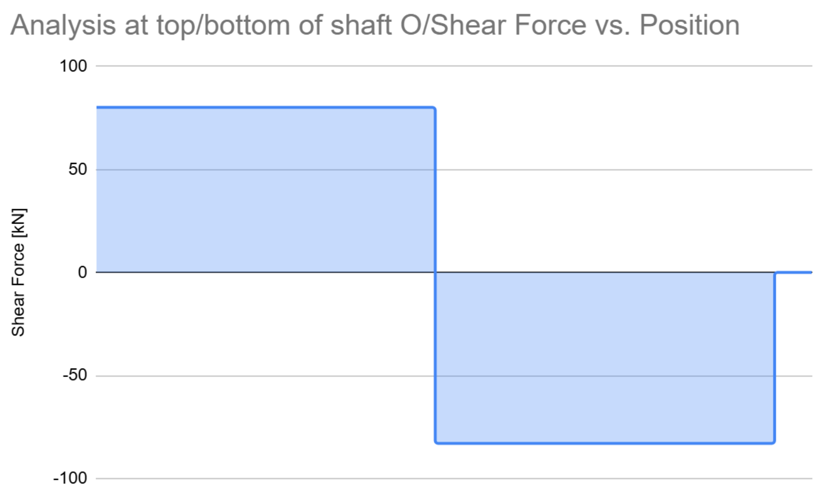


Figure 6.26 Shaft O Shear force diagram for Horizontal forces, used in analysis at top/bottom of shaft

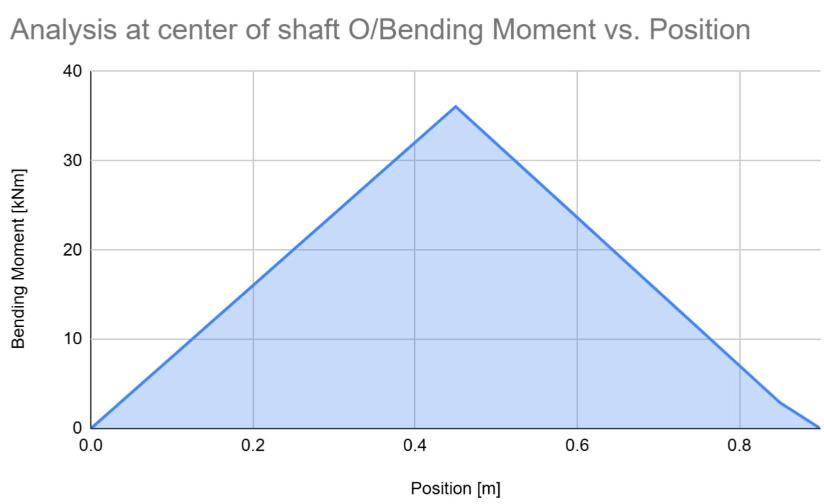


Figure 6.27 Shaft O Bending Moment diagram for Horizontal forces, used in analysis at center of shaft

And we can develop the following tables to determine the safety factor for each critical location:

Table 6.14 Shear forces, Bending Moments, and stress states at top of shaft O

Critical Location	V j^ [kN]	M -j^ [kNm]	T i^ [kNm]	Normal Stress [MPa]	Shear Stress [MPa]
Centre of Left Bearing	80.18	0.00	0.00	0.00	44.14
First Bearing Shoulder	80.18	-8.55	0.00	2.22	44.14
Gear Shoulder	80.18	-17.11	-331.57	2.35	23.56
Centre of Gear	-82.94	-76.99	-331.57	10.59	23.59
Second Bearing Shoulder	-82.94	-8.82	-331.57	2.29	44.18
Centre of Right Bearing	-82.94	0.00	-331.57	0.00	44.18
Coupler Keyway	0.00	0.00	-331.57	0.00	51.53

Table 6.15 Shear forces, Bending Moments, and stress states at sides of shaft O

Critical Location	V k^ [kN]	M k^ [kNm]	Normal Stress [MPa]	Shear Stress [MPa]
Centre of Left Bearing	-171.10	0.00	0.00	45.48
First Bearing Shoulder	-171.10	4.01	2.22	45.48
Gear Shoulder	-171.10	8.02	2.35	24.44
Centre of Gear	187.29	36.08	10.59	24.60
Second Bearing Shoulder	176.45	4.15	2.29	45.56
Centre of Right Bearing	176.45	0.00	0.00	45.56
Coupler Keyway	0.00	0.00	0.00	51.53

Table 6.16 Stress Concentrations for Shaft O

Critical Location	Maximum Nominal Principal Stress [MPa]	Kt	q	Kf	Maximum Principal Stress [MPa]
Centre of Left Bearing	78.77	1.000	0.959	1.000	78.77
First Bearing Shoulder	78.78	1.649	0.985	1.639	129.14
Gear Shoulder	42.34	2.289	0.972	2.253	95.39
Centre of Gear	42.89	2.750	0.959	2.678	114.86
Second Bearing Shoulder	78.91	1.729	0.982	1.716	135.43
Centre of Right Bearing	78.91	1.000	0.982	1.000	78.91
Coupler Keyway	89.26	2.650	0.959	2.582	104.89

Table 6.17 Material Limits and Safety Factors for Shaft O

Critical Location	diameter at point [m]	Maximum Principal Stress [MPa]	Material limit at location [MPa]	Safety Factor
Centre of Left Bearing	0.34	78.77	192.85	2.45
First Bearing Shoulder	0.34	129.14	192.85	1.49
Gear Shoulder	0.42	95.39	192.85	2.02
Centre of Gear	0.42	114.86	192.85	1.68
Second Bearing Shoulder	0.34	135.43	192.85	1.42
Centre of Right Bearing	0.34	78.91	192.85	2.44
Coupler Keyway	0.32	104.81	192.85	1.84

6.5 Detailed Gear Calculations

6.5.1 General Gear Specifications

The gear calculations have been done considering spur gears with a module of 25mm. The number of teeth on each gear were then selected such that the minimum number of teeth, which is 18 for a pressure angle of 20° [1], was exceeded, and such that the gear diameters would not be unreasonable large.

$$Module = m = \frac{d}{N} = \frac{2r}{N} \rightarrow r = \frac{mN}{2}$$

$$W_T = \frac{T_P}{r_P}$$

$$W_r = W_T \tan\phi$$

Where N is the number of teeth, m is the module, r is the pitch radius, W_T is the tangential force, W_r is the radial force, T_p is the torque of the pinion, r_p is the pitch radius of the pinion, and φ is the pressure angle. Table 7.18 summarizes these calculations.

Table 6.18 Number of teeth, pitch radius, angular frequency, torque, tangential force, and radial force for all gears

Gear	Number of Teeth, N	Pitch Radius, r (m)	Angular frequency, w (rad/s)	Torque, T (kNm)	Tangential force, W_T (kN)	Radial Force, W_r (kN)
1	20	0.25	628	25.904	-95.941	37.713
2	40	0.5	-314	-51.108	95.941	-37.713
3	20	0.25	-314	-51.108	-189.289	-74.407
4	40	0.5	157	100.835	189.289	74.407
5	18	0.225	157	100.835	-480.165	163.115
6	60	0.75	-47	331.572	480.165	-163.115

6.5.2 Gear Stresses

The table below outlines the bending (N_b) and Surface (N_c) safety factors for each gear, based on the material limits for each gear, as well as on their stress states. The specifications are presented in table 7.19 below. See calculations for table 7.19 in appendix B.

Table 6.19 Face width, bending and surface stress, infinite life and safety factor, for all gears

Gear	Face Width, F (m)	Bending Stress, σ_b (MPa)	Surface Stress, σ_b (MPa)	Bending Infinite life, S_{fb} (MPa)	Surface Infinite life, S_{fc} (MPa)	Bending Safety Factor, N_b	Surface Safety Factor, N_c
1	0.2	167.15	830.23	336.78	1037.26	2.01	1.56
2	0.2	153.59	830.23	344.41	1053.93	2.24	1.61
3	0.4	185.59	874.84	344.01	1053.93	1.86	1.45
4	0.4	170.54	874.84	352.20	1070.86	2.07	1.50
5	0.7	164.23	905.04	352.20	1070.86	2.14	1.50
6	0.7	138.67	905.04	366.17	1100.93	2.64	1.48

6.5.3 Tooth Specifications

$$p_d = \frac{25.4}{m} = 1.016in^{-1}$$

Calculations for gear tooth specifications were done using figure 7.28 [1] below. The location of each parameter can be seen in figure 7.29[1].

Table 12-1 AGMA Full-Depth Gear Tooth Specifications

Parameter	Coarse Pitch ($p_d < 20$)	Fine Pitch ($p_d \geq 20$)
Pressure angle ϕ	20° or 25°	20°
Addendum a	1.000 / p_d	1.000 / p_d
Dedendum b	1.250 / p_d	1.250 / p_d
Working depth	2.000 / p_d	2.000 / p_d
Whole depth	2.250 / p_d	2.200 / p_d + 0.002 in
Circular tooth thickness	1.571 / p_d	1.571 / p_d
Fillet radius—basic rack	0.300 / p_d	not standardized
Minimum basic clearance	0.250 / p_d	0.200 / p_d + 0.002 in
Minimum width of top land	0.250 / p_d	not standardized
Clearance (shaved or ground teeth)	0.350 / p_d	0.350 / p_d + 0.002 in

Figure 6.28 Gear specifications for standard Spur gears [1]

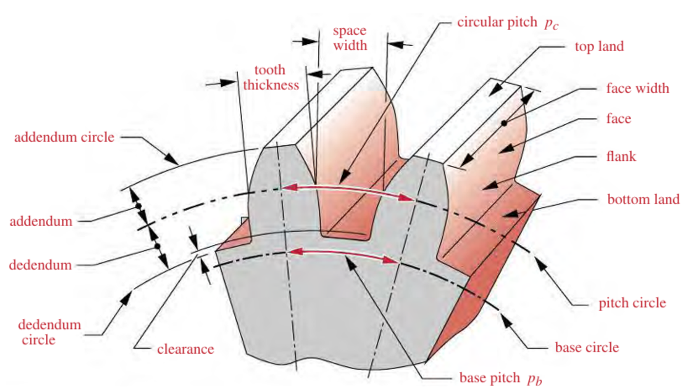


Figure 6.29 Standard Gear terminology [1]

The resulting specifications can be seen below in table 7.20

Table 6.20 Gear parameter dimensions

Parameter	Size (mm)
Addendum (a)	25
Dedendum	31.25

Working Depth	50
Whole Depth	56.25
Circular Tooth thickness	39.275
Fillet radius for basic rack	7.5
Minimum basic clearance	6.25
Minimum width of top land	6.25
Estimated tooth height	62.5

6.5.4 Gear mesh contact ratios

We must also make sure that a minimum (m_p) contact ratio of 1.4 is achieved [1]. Table 7.21 below outlines the contact ratios of our gear meshes, where C is the center-to-center distance, and Z is the length of action. See Appendix B for detailed calculations of table 7.21.

Table 6.21 Center-to-center distance, length of action, and contact ratio for each gear mesh

Gear mesh	C (m)	Z (m)	Contact Ratio, m_p
1,2	0.81	0.100	1.70
3,4	0.81	0.100	1.70
5,6	0.91	0.0997	1.69

6.6 Fixation to the housing

All bearings must be fixed to the shaft and housing appropriately, ensuring one side is floating, and the other is fixed, for each shaft. Axial forces are minimal, so simple c-clips may be used to fix floating bearings to the shafts. Further justification for this choice may be found in a previous section. All floating bearings, except the right bearing on Shaft I, which floats within the bearing itself. Lastly, spacers will be needed between the bearings and their shoulders to avoid clamping both sides to the shaft.

Figure 7.30 shows the arrangement for the left side of Shaft I, which is mirrored for the right side of shaft O. Note that Shaft O uses a spherical roller bearing instead. Figure 7.31 shows how the coaxial shafts, I and B, will be held. Figure 7.32 shows the right side of Shaft B, which is mirrored for the left side of shaft A. Figure 7.33 shows the left side of shaft O, which is mirrored for the right side of shaft A.

Appendix C provides more detail about the bearings used.

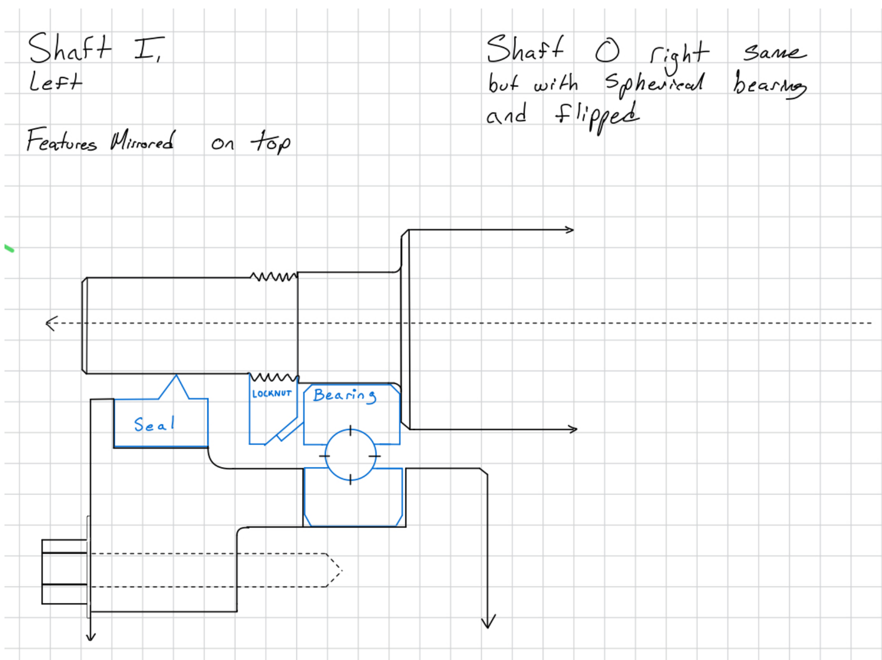


Figure 6.30 Shaft I and O (mirrored)

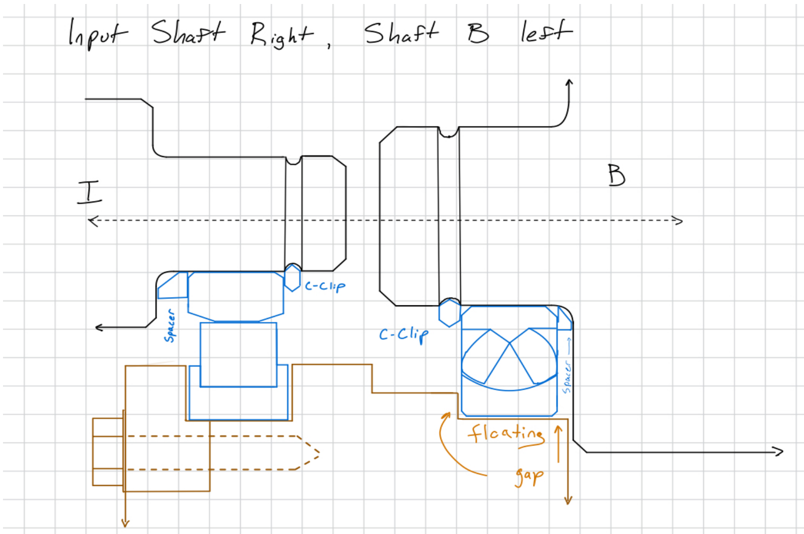


Figure 6.31 Coaxial shafts I and B

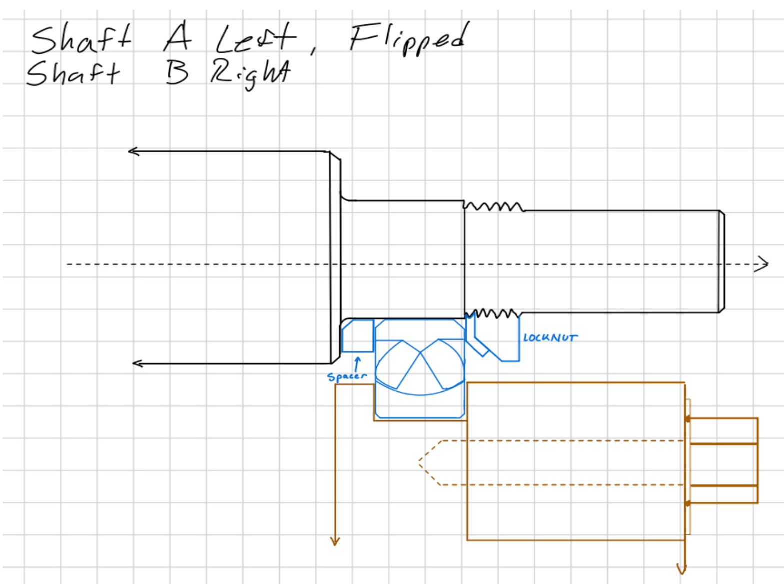


Figure 6.32 Shaft A (mirrored) and B

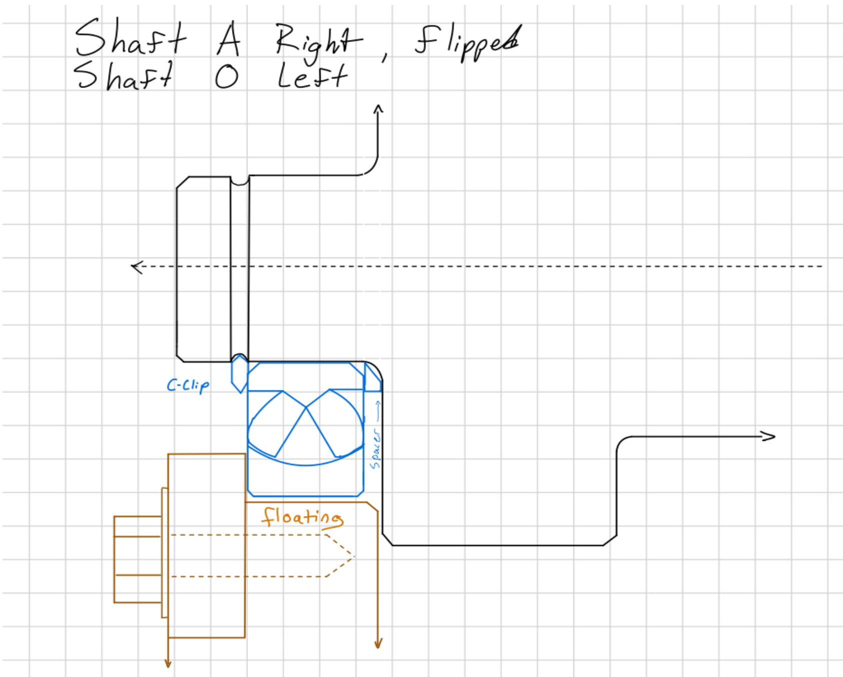


Figure 6.33 Shaft A (mirrored) and O

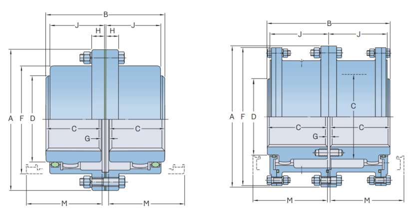
6.7 Keyways

All keyways used were end-milled and standard sizes [16]. All keys used were made of 1060 hot-rolled carbon steel. Keys are relatively inexpensive and easy to replace thus the key material was chosen to have a "lower strength than that of the shaft so that a bearing failure will selectively affect the key rather than the keyway if the system sees an overload beyond its design range.[15]"

6.8 Couplings

Power is transferred in and out of the input and output shaft respectively through the use of double-engagement gear couplings. The couplings attach to the shafts using keys which are of standardized dimension [16]. Gear couplings were chosen due to their high torque and rpm capabilities. The specific bearing chosen for the input shaft is the *SKF Double Engagement 40GC* and for the output the *SKF Double Engagement 110GC*. The full specifications of these couplings can be seen in Figure 7.34 below.

Double engagement



Size 10 to 70

Size 80 to 200

Size	Power per 100 r/min	Rated torque	Speed	Bore diameter		Dimensions								Gap	Lubricant weight	Coupling weight without bore
			Max.	Min.	Max.	A	B	C	D	F	H	J	M[1)]	G Min.		
–	kW	Nm	r/min	mm										mm	kg	
10 GC	11.9	1 139	8 000	13	50	116	89	43	69	84	14	39	51	3	0.04	5
15 GC	24.6	2 350	6 500	20	65	152	101	49	86	105	19	48	61	3	0.07	9
20 GC	44.7	4 270	5 600	26	78	178	127	62	105	126	19	59	77	3	0.12	16
25 GC	78.3	7 474	5 000	32	98	213	159	77	131	155	21.8	72	92	5	0.23	29
30 GC	127	12 100	4 400	38	111	240	187	91	152	180	21.8	84	107	5	0.36	43
35 GC	194	18 500	3 900	51	134	279	218	106	178	211	28.4	98	130	6	0.54	68
40 GC	321	30 609	3 600	64	160	318	248	121	210	245	28.4	111	145	6	0.91	97
45 GC	440	42 000	3 200	77	183	346	278	135	235	274	28.4	123	166	8	1.04	136
50 GC	593	56 600	2 900	89	200	389	314	153	254	306	38.1	141	183	8	1.77	190
55 GC	775	74 030	2 650	102	220	425	344	168	279	334	38.1	158	204	8	2.22	249
60 GC	947	90 400	2 450	115	244	457	384	188	305	366	25.4	169	229	8	3.18	306
70 GC	1 420	135 000	2 150	127	289	527	452	221	343	425	28.4	196	267	10	4.35	485
80 GC	1 780	170 000	1 750	102	266	591	508	249	356	572	–	243	300	10	9.53	703
90 GC	2 360	226 000	1 550	115	290	660	565	276	394	641	–	265	327	13	12.25	984
100 GC	3 250	310 000	1 450	127	320	711	623	305	445	699	–	294	356	13	14.97	1 302
110 GC	4 320	413 000	1 330	140	373	775	679	333	495	749	–	322	384	13	17.69	1 678
120 GC	5 810	555 000	1 200	153	400	838	719	353	546	826	–	341	403	13	20.87	2 114
130 GC	7 528	719 000	1 075	165	440	911	762	371	584	886	–	367	435	19	32.65	2 595
140 GC	9 539	911 000	920	175	460	966	806	393	635	940	–	378	457	19	33.10	3 107
150 GC	11 518	1 100 000	770	190	490	1 029	857	419	686	1 003	–	408	483	19	40.81	3 765
160 GC	13 715	1 310 000	650	250	525	1 111	908	441	736	1 086	–	419	502	25	43.08	4 708
180 GC	17 382	1 660 000	480	285	600	1 219	939	457	838	1 194	–	435	521	25	49.90	6 260
200 GC	22 406	2 140 000	370	315	660	1 359	1 099	537	927	1 306	–	514	635	25	68.00	8 582

1) Minimum clearance required for aligning coupling.
Double engagement couplings are designed for most horizontal, close coupled applications. This coupling accommodates both offset and angular misalignment, as well as end float.
Applications include: fans, pumps, steel and paper mill drives, cranes and conveyors.
Bore tolerances will be K7 and key width (b) will be P9 (close fit) for coupling sizes 130 and bigger unless stated otherwise.
Weights are given, in kg, with minimum listed bore, excluding lubricant.

30 SKF

Figure 6.34 Double Engagement Coupler Spec Sheet[17]

6.9 Fasteners

Fasteners are important in securing the bearings within the gearbox housing. The housing serves as gearbox's structural backbone, supporting shafts and keeping them aligned under high torque and fluctuating loads. Fasteners are essential for preventing bearing misalignment and ensuring the housing's structural integrity under operational stress.

The fasteners securely attach the bearings that are between the two halves of the housing. These fasteners are installed in two rows on either side of the housing and are designed to handle the axial and radial forces transmitted through the bearings, as well as the fluctuating loads generated when the gearbox is in operation.

The housing halves are bolted together using the M36 socket head screws once the bearings have been inserted in their designated locations. These screws can withstand forces exerted by the shafts and bearings, ensuring that the gearbox remains structurally reliable.

Additionally, the housing design introduces features to facilitate easy fabrication. Confined gaskets are used along the joint surfaces to prevent lubricant leakage, which is important for maintaining the efficiency and longevity of gearbox components. The M36 fasteners not only provide the necessary clamping force to secure the bearings but also allow for disassembly when maintenance is needed.

The use of M36 screws, supported by our extensive calculations, assures that the housing and bearings can resist operational demands during the gearbox's intended 25-year life period.

The table below covers calculations such as external forces, preload stress, and mean stress, which helped with our fastener selection. These calculations ensure that the fasteners are the proper size and configuration to attach the bearings to the housing.

Table 6.22 Fasteners Calculations

Shafts	Preload, Fi k(N)	Max External load $P_{max\ (kN)}$	Bolt Force (kN)	Alternating force (N)	Mean force (N)	Alternating stress (MPa)	Mean stress (MPa	Preload stress (MPa)	Nf	$N_{yielding}$
Input	594.16	12.63	596.78	1.31	595.47	10.67	729.10	727.50	8.76	1.51
Output	594.16	42.66	603.00	4.42	598.58	36.05	732.91	727.50	2.59	1.49

Based on the calculations above, we chose the **McMaster M36x4 – 160 Socket head screws (PN-**91290A898) as fasteners to secure the bottom bearings to the gearbox housing. The drawing of these fasteners is found in the CAD Drawings section of our report.

6.10 Bearings

Bearings are critical components in ensuring smooth and efficient rotational movement within the gearbox. They support the shafts, reduce friction between moving parts, and maintain alignment under the high stresses of operational loads. The proper selection and placement of bearings directly impact the gearbox's performance, durability, and efficiency.

The table below details the calculations used to determine bearing load capacities, dynamic life, and fit tolerances. These calculations were vital in ensuring the selected bearings could withstand operational stresses and maintain precision in the gearbox assembly.

Table 6.23 Bearing Lifetime Calculations

Bearing LOC	**Bearing**	**Type**	**Dynamic Load Rating (C) [kN]**	**Static Load Rating (C_0) [kN]**	**Reference Speed (Oil) [RPM]**
Input Shaft, 1	Timken 6028	Deep Groove Ball Bearing	110	101.8	5600
Input Shaft, 2	NU 222 ECML Single row cylindrical roller bearing, NU design	NU Single Row Roller Bearing	335	101.8	3600
Shaft A, 1	Timken 22218EJW33C3	Spherical Roller Bearing	355	388	4300
Shaft A, 2	Timken 22218EJW33C3	Spherical Roller Bearing	355	388	4300
Shaft B, 1	Timken 23024EJW33	Spherical Roller Bearing	408	574	3300
Shaft B, 2	Timken 23024EJW33	Spherical Roller Bearing	408	574	3300
Output Shaft, 1	Timken 23968EMBW33	Spherical Roller Bearing	1670	2990	990
Output Shaft, 2	Timken 23968EMBW33	Spherical Roller Bearing	1670	2990	990

…

Table Continued Below

RPM	Radial Load Factor, X	Radial Load [kN]	Number of Bearings	Equivalent Dynamic load, P [kN]	Lifetime [hours]	Lifetime [Days]	Lifetime [Months]	Lifetime [Years]	Maintenances [Months]
6000	0.44	50.846	4	5.593	21130.917	880.455	29.348	2.412	24
6000	0.23	62.806	1	14.445	98799.268	4116.636	137.221	11.278	132
3000	0.67	55.091	1	36.911	10510.748	437.948	14.598	1.200	12
3000	0.67	75.740	2	25.373	36665.107	1527.713	50.924	4.186	48
1500	0.67	109.750	2	36.766	33867.975	1411.166	47.039	3.866	36
1500	0.67	320.294	4	53.649	9610.527	400.439	13.348	1.097	12
450	0.67	188.951	1	126.597	200882.864	8370.119	279.004	22.932	240
450	0.67	194.970	1	130.630	180947.216	7539.467	251.316	20.656	240

6.10.1 Bearing Maintenance Schedule

When operating this device, the life of each individual part, shaft, gear, and bearing must be carefully considered. Failing to replace bearings at the end of their calculated lifespan can result in several risks and hazards, including excessive wear, misalignment, damage to the surrounding components and ultimately gearbox failure. Neglecting the following maintenance schedule may lead to unplanned downtime, higher repair costs, and safety hazards.

The following schedule outlines the planned replacement intervals for all bearings. Aligned maintenance tasks are included to optimize efficiency and reduce downtime. Bearings with shorter lifespans are replaced during overlapping intervals with those of longer lifespans to ensuring minimal disruption.

Table 6.24 Bearing Maintenance Schedule

Interval (Months)	Bearings to Replace	Notes
12	• Shaft A Bearing 1 (Timken 22218EJW33C3) • Shaft B Bearing 2 (Timken 23024EJW33)	Bearings with Shortest Lifespan
24	• Input Shaft Bearing 1 (Timken 6028) • Shaft A Bearing 1 (Timken 22218EJW33C3) • Shaft B Bearing 2 (Timken 23024EJW33)	Combine replacements with 12-month bearings to minimize downtime.
36	• Shaft B Bearing 1 (Timken 23024EJW33), • Shaft A Bearing 1 (Timken 22218EJW33C3), • Shaft B Bearing 2 (Timken 23024EJW33), • Input Shaft Bearing 1 (Timken 6028)	Replace all bearings with 12- and 24-month lifespans alongside 36-month bearings for operational efficiency.
48	• Shaft A Bearing 2 (Timken 22218EJW33C3), • Shaft A Bearing 1 (Timken 22218EJW33C3), • Shaft B Bearings 1 and 2 (Timken 23024EJW33), • Input Shaft Bearing 1 (Timken 6028)	Replace all bearings with 12-, 24-, and 36-month lifespans during this interval.
132	• Input Shaft Bearing 2 (NU 222 ECML), • Shaft A Bearings 1 and 2 (Timken 22218EJW33C3), • Shaft B Bearings 1 and 2 (Timken 23024EJW33), • Input Shaft Bearing 1 (Timken 6028)	Comprehensive replacement of all bearings ensures continued reliability for the next several years.
240	• Output Shaft Bearings 1 and 2 (Timken 23968EMBW33), • Input Shaft Bearings 1 and 2 (Timken 6028, NU 222 ECML), • Shaft A Bearings 1 and 2 (Timken 22218EJW33C3), • Shaft B Bearings 1 and 2 (Timken 23024EJW33)	Full gearbox overhaul, replacing all bearings simultaneously to extend system lifespan and reliability.

6.11 Assembly Sequence

Each shaft assembly will follow a similar order of assemblance. To illustrate, we can examine the step-by-step assembly of the input shaft. Firstly, the keystock will be inserted into the shafts keyway, where then the gear will be mounted. Next, we can hold the gear in place by using a doubly bolted compression fitting collar (This is mirrored for shafts with two gears).

Further, once the shafts are assembled, we will insert each shaft into its respective bearing set, and place c-clamps, locknuts and washers on the shafts. Seals are also slid into place on the input and output shafts.

Once each shaft has all its features properly located, each shaft will be mounted onto the bottom housing. The top housing will be placed on top, and the fasteners holding these will be placed and tightened to snug + ¼ turn. Lubrication can now be added.

Couplings may now be attached, and the gearbox assembly mounted onto its base and connected to the turbine and generator.

Specifications for bearing nuts and their respective lock washers, collars, and seals can be found in appendix C.

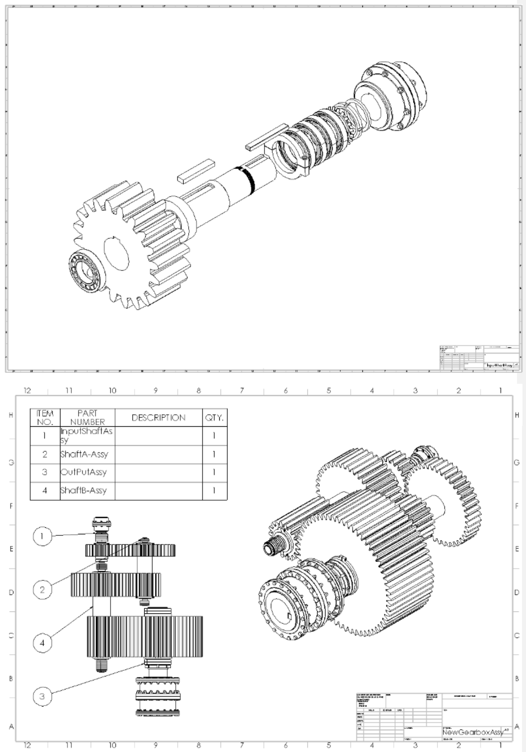


Figure 6.36 Gearbox Assembly

7 Housing

The housing for the gearbox has been redesigned, such that it is only composed of two pieces plus fasteners. It is now designed such that no holding plates are required to fix the bearings to the housing, and such that any previous concept drawings for the housing are still valid, though it will all be in one piece above and one below. The bearings are hence sandwiched between the two pieces of the housing. Likewise, a confined gasket around the edges of the housing at the joint is integrated into the design to prevent the grease from leaking.

Furthermore, the housing has one hole at the bottom to allow oil to be drained, and one at the top to allow oil to be pumped in.

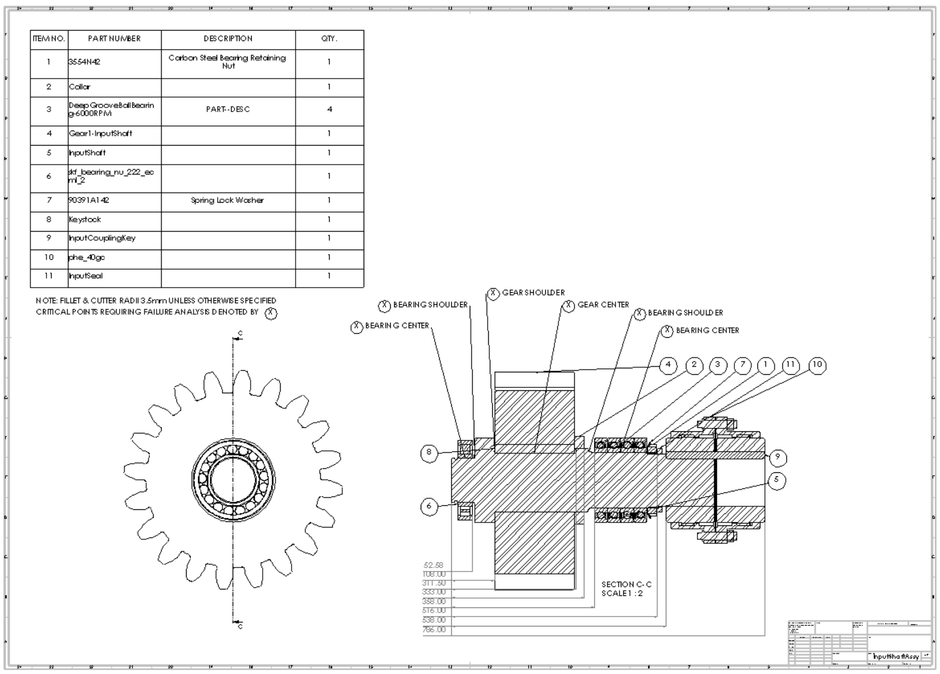
The top and bottom halves of the housing are aligned using dowel pins near the bearings, and fastened together using two rows of 9 M24x3 – 110 bolts. These rows are located at the sides of the assembly.

As specified in section 7.9 above, the housing will be attached to its bottom plate using 2 rows of 9 M36x4-160 screws.

8 CAD Drawings

Commented [AA1]: Need drawings for housing and full housing assmbly

8.1 Input Shaft



ITEM NO.	PART NUMBER	DESCRIPTION	QTY.
1	3554N42	Carbon Steel Bearing Retaining Nut	1
2	Collar		1
3	DeepGrooveBallBearing-6000RPM	PART--DESC	4
4	Gear1-InputShaft		1
5	InputShaft		1
6	skf_bearing_nu_222_ecml_2		1
7	90391A142	Spring Lock Washer	1
8	Keystock		1
9	InputCouplingKey		1
10	phe_40gc		1
11	InputSeal		1

Figure 8.1 Input Shaft Assembly

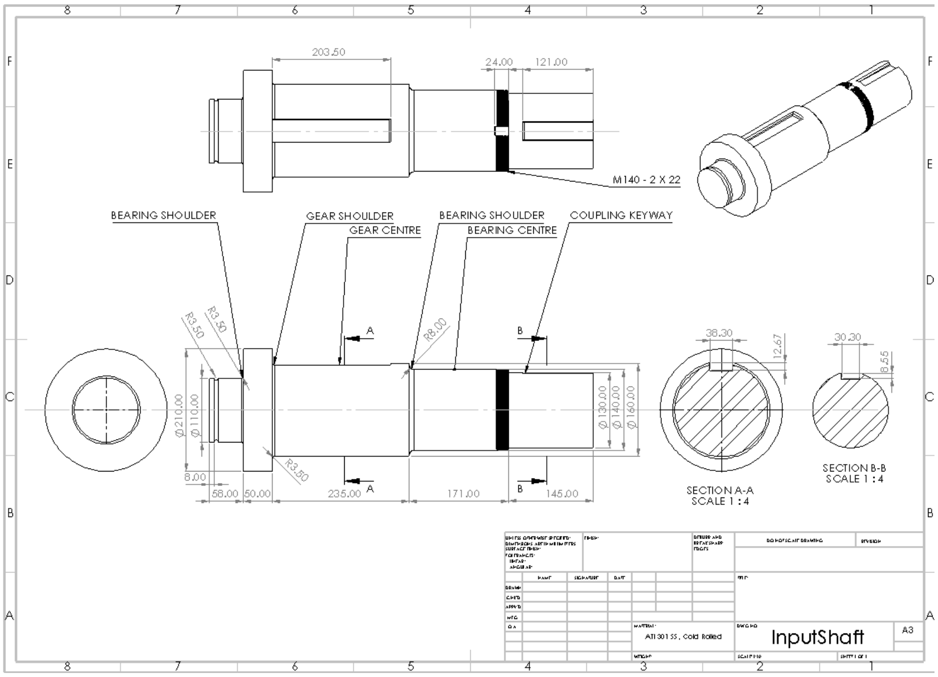


Figure 8.2 Input Shaft

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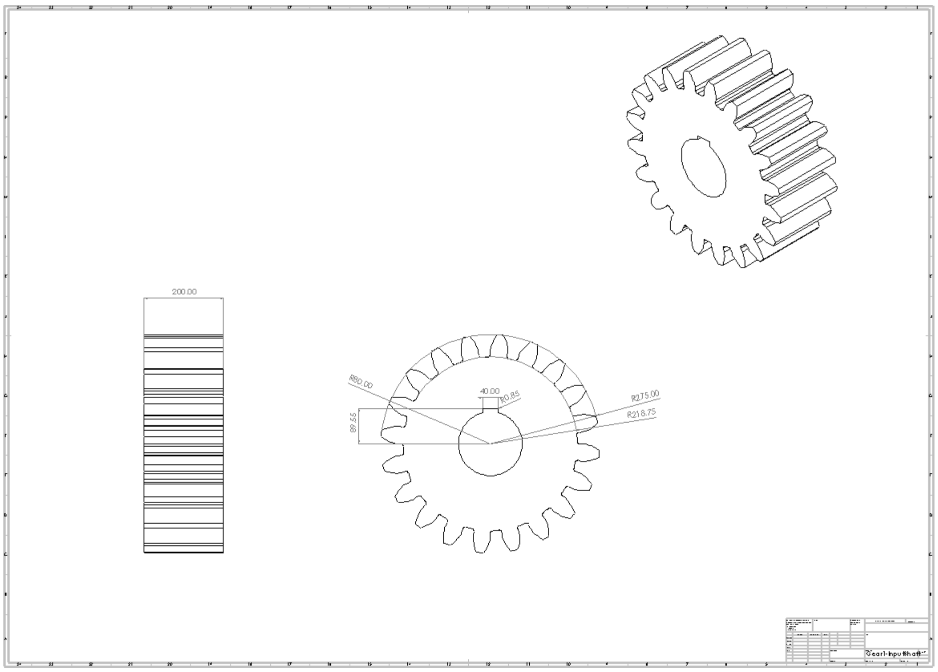
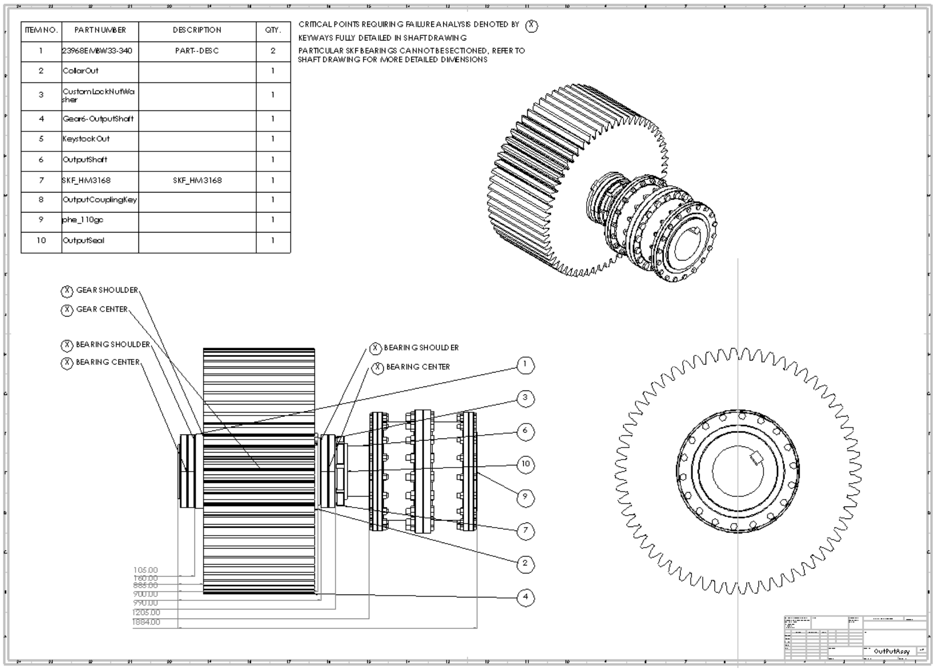


Figure 8.3 Gears for Input Shaft

8.2 Output Shaft



ITEM NO.	PART NUMBER	DESCRIPTION	QTY.
1	23968EMBW33-340	PART--DESC	2
2	CollarOut		1
3	CustomLockNutWasher		1
4	Gear6-OutputShaft		1
5	KeystockOut		1
6	OutputShaft		1
7	SKF_HM3168	SKF_HM3168	1
8	OutputCouplingKey		1
9	phe_110gp		1
10	OutputSeal		1

Figure 8.4 Output Shaft Assembly

Commented [AA3]: Drawing needs updating

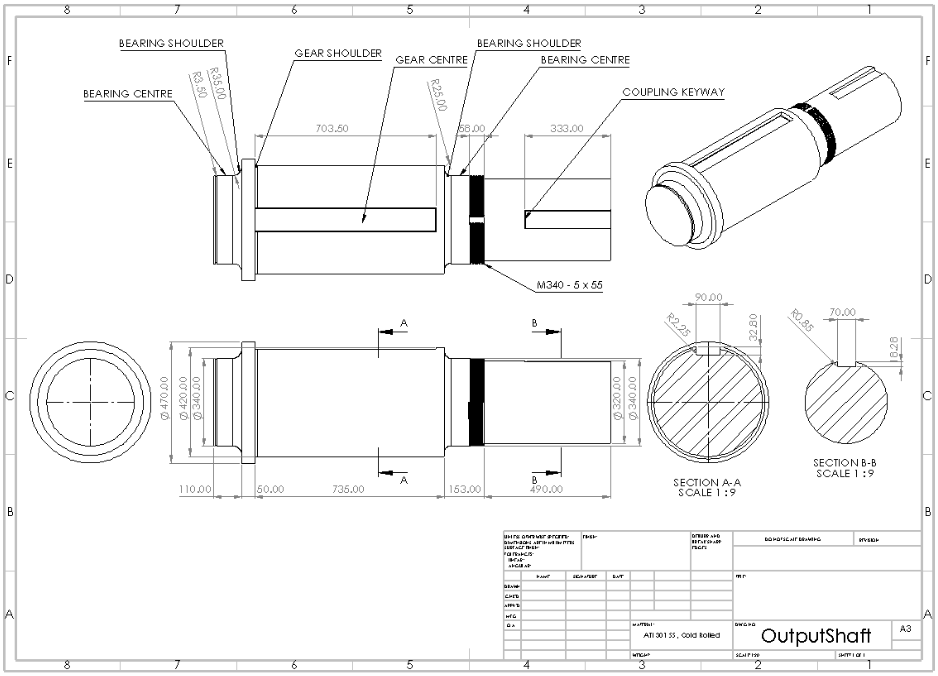


Figure 8.5 Output Shaft

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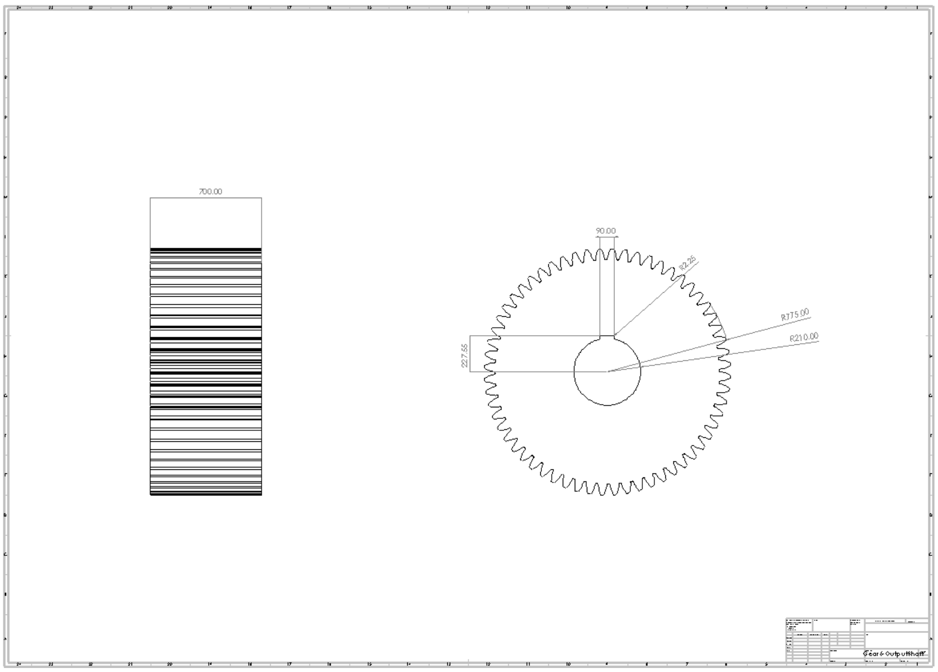

Figure 8.6 Gear for Output Shaft

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8.3 Shaft A

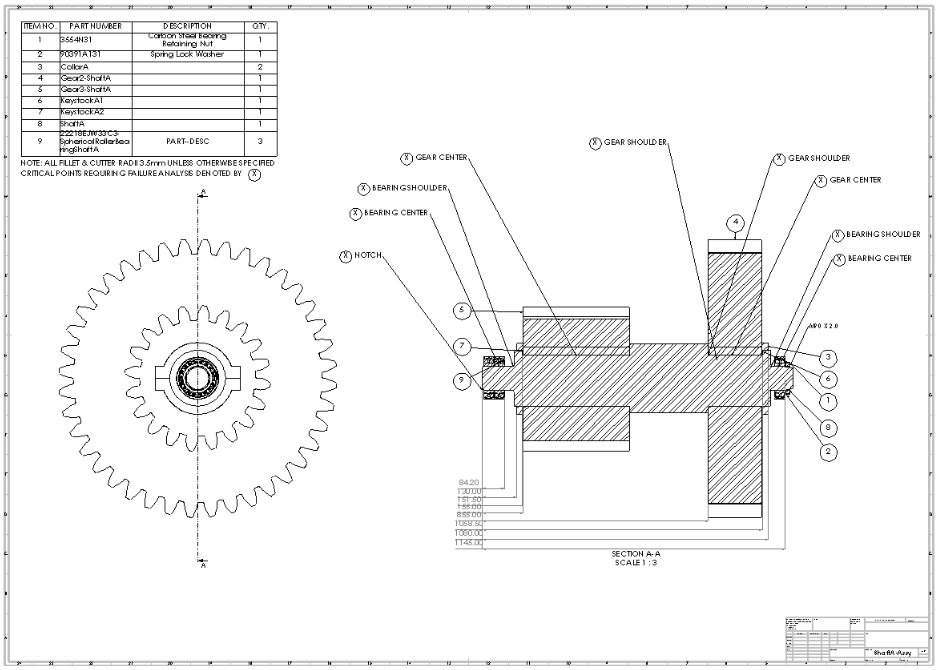

Figure 8.7 Shaft A Assembly

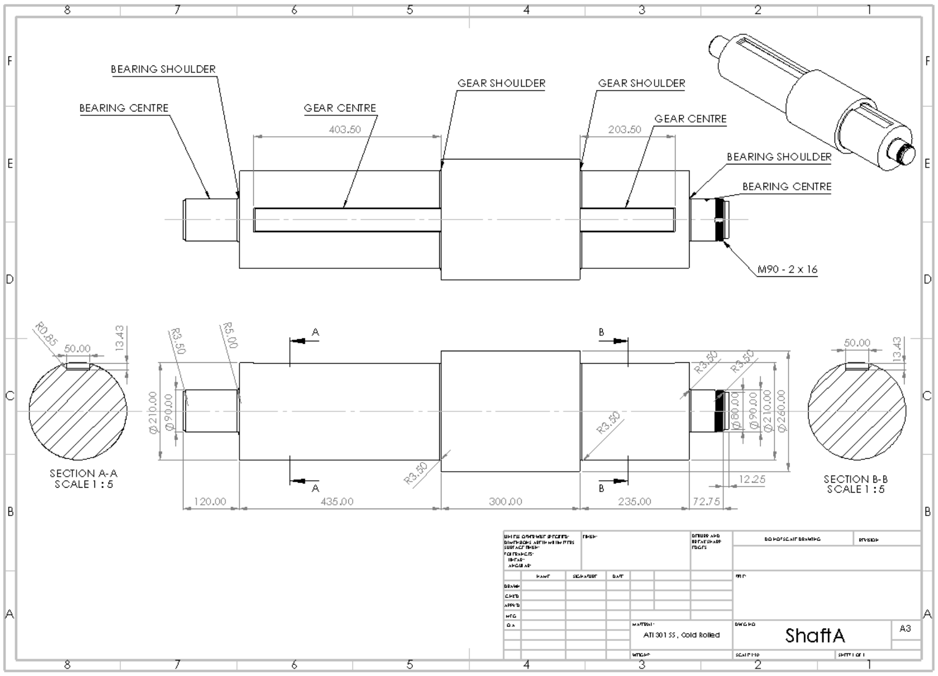

Figure 8.8 Shaft A

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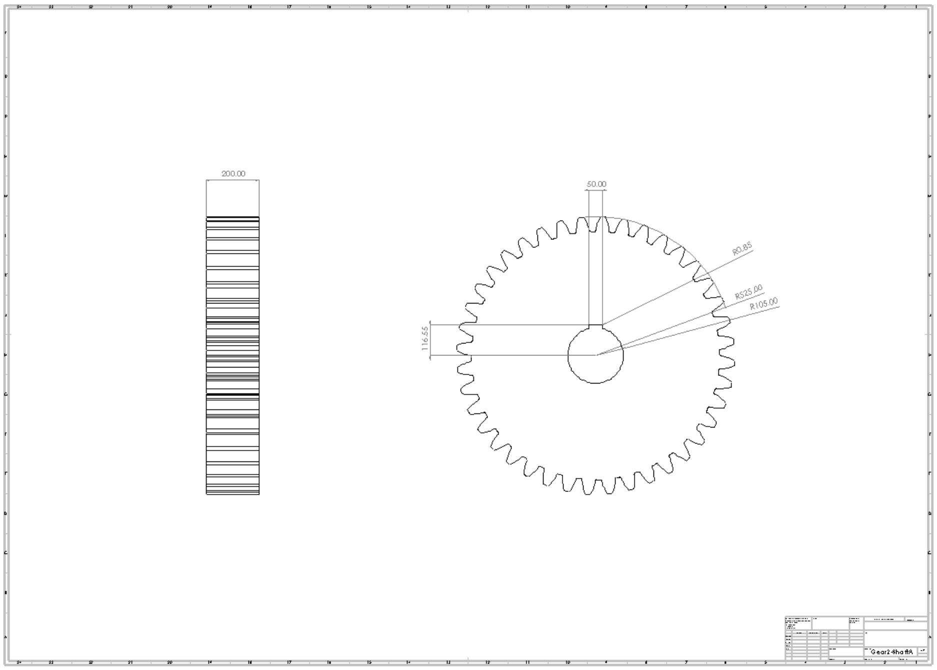

Figure 8.9 Gear 2, Shaft A

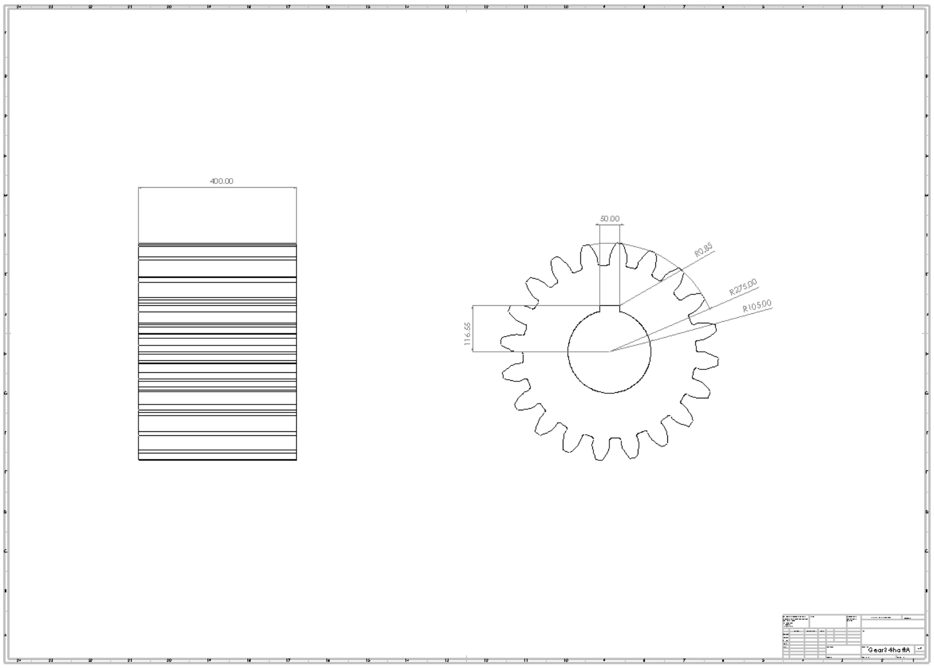

Figure 8.10 Gear 3, Shaft A

8.4 Shaft B

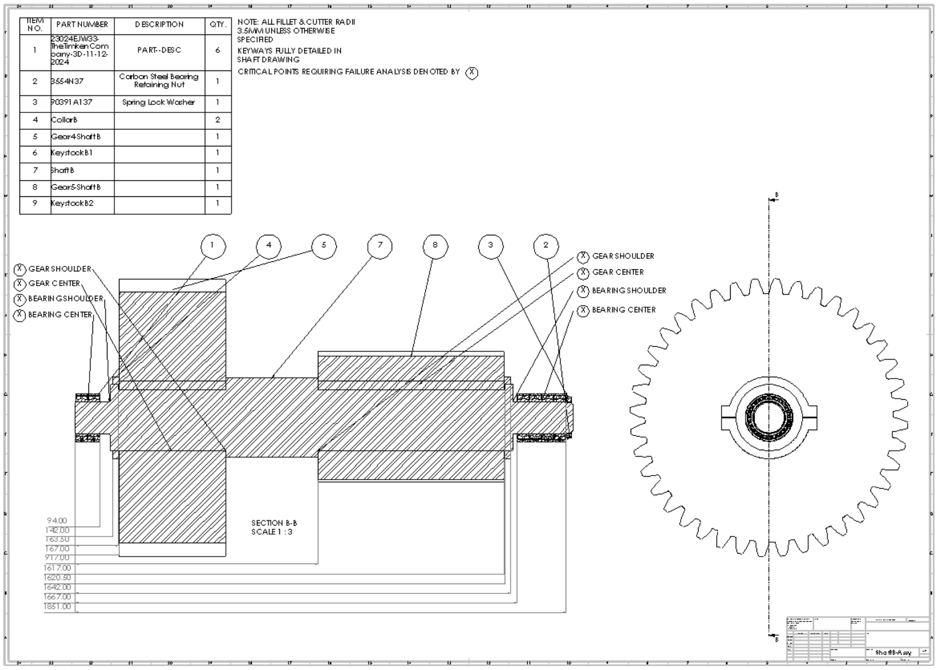

Figure 8.11 Shaft B Assembly

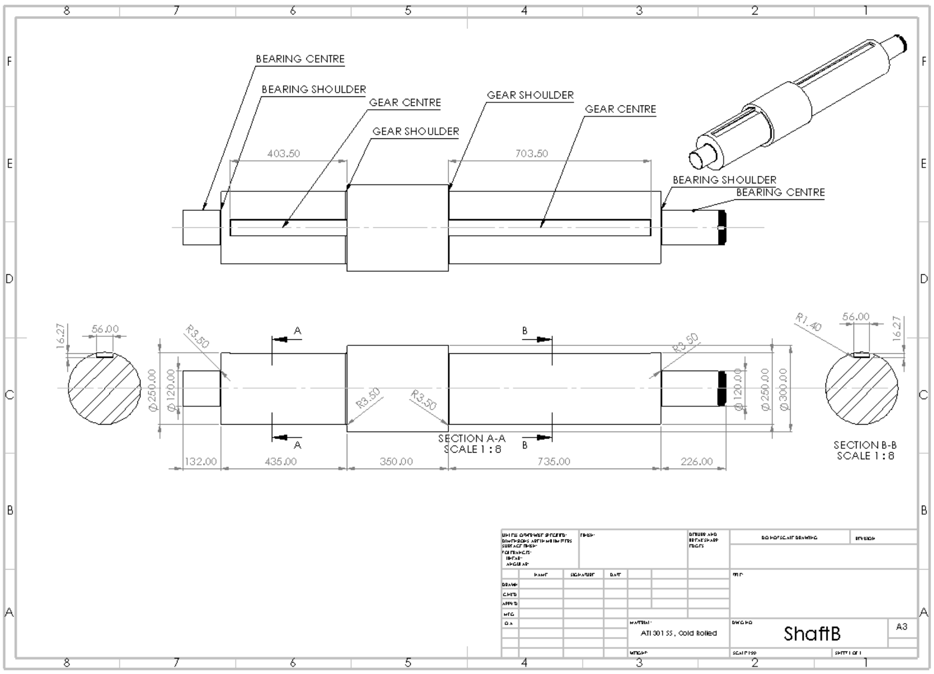

Figure 8.12 Shaft B

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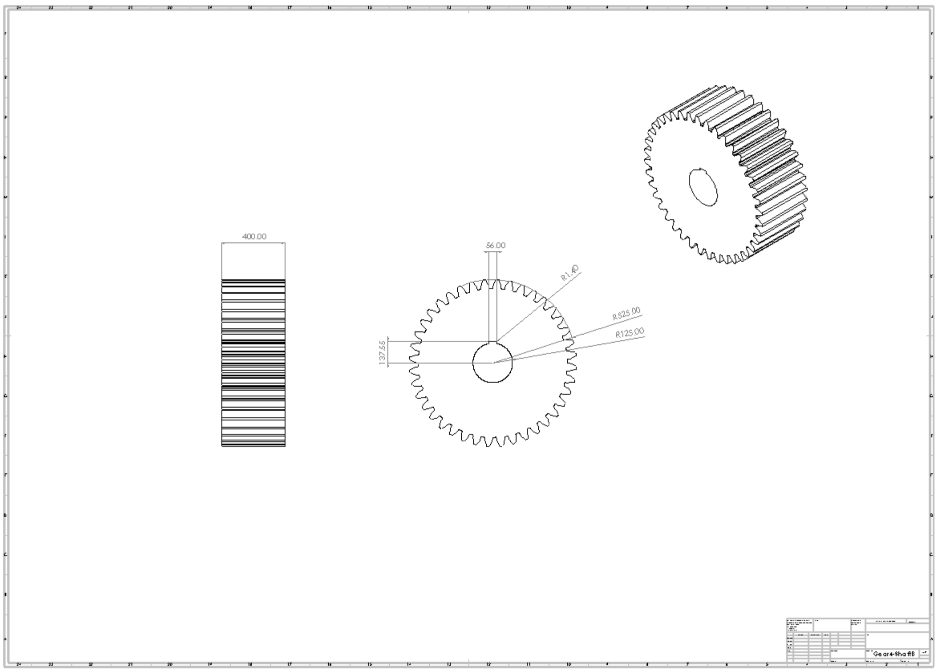

Figure 8.13 Gear 4, Shaft B

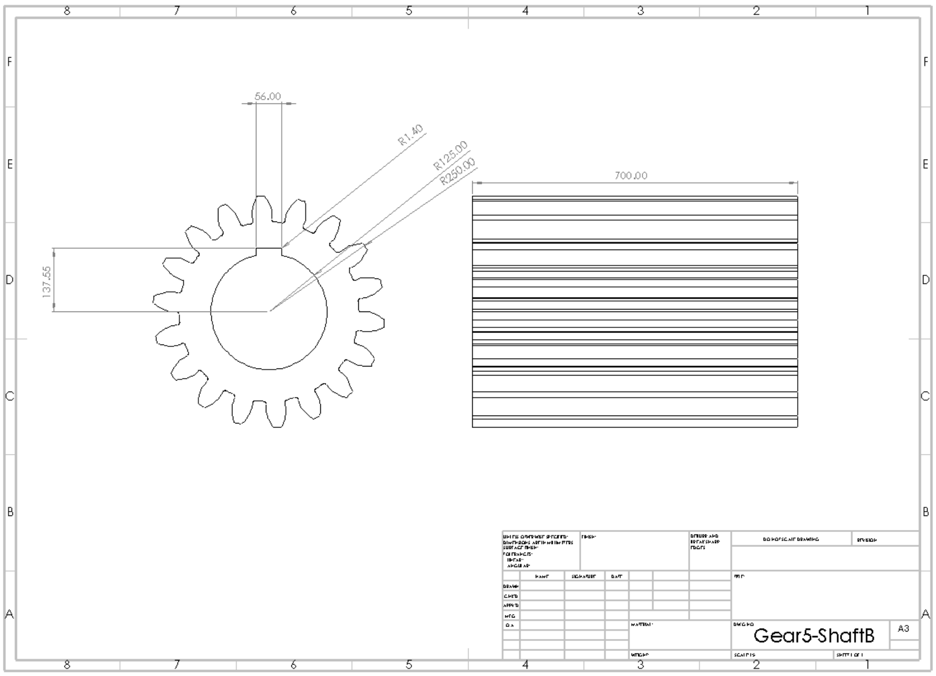

Figure 8.14 Gear 5, Shaft B

8.5 Housing

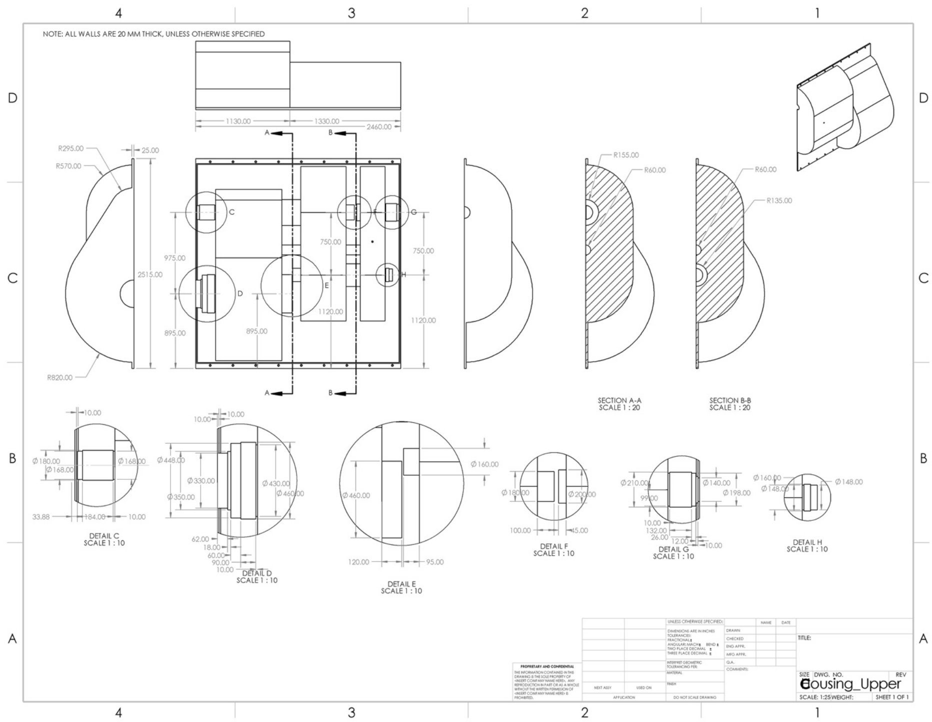

Figure 8.15 Housing Upper

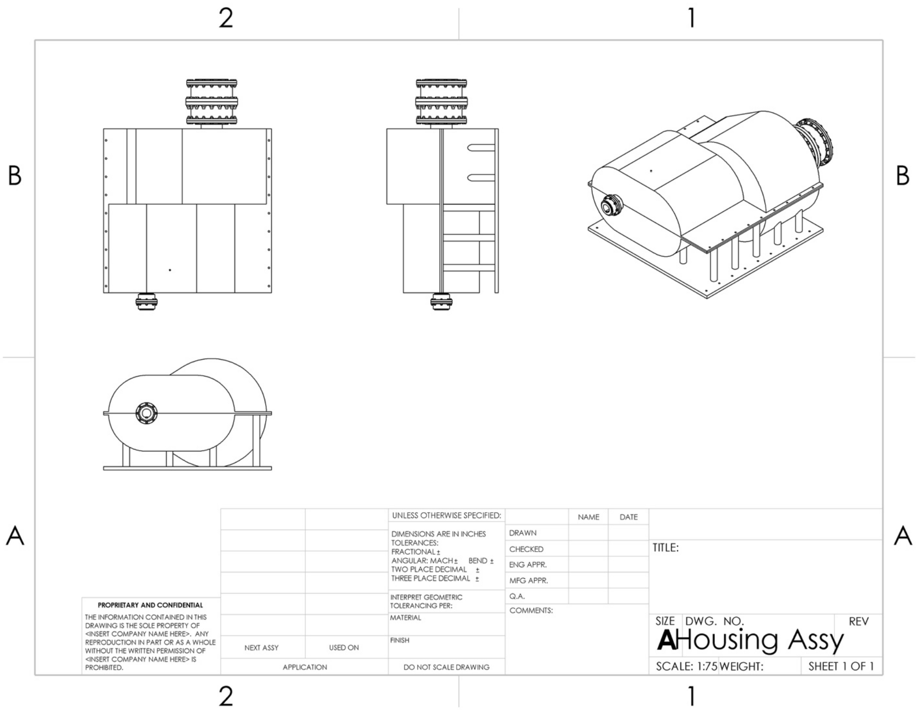

Figure 8.16 Housing Assembly

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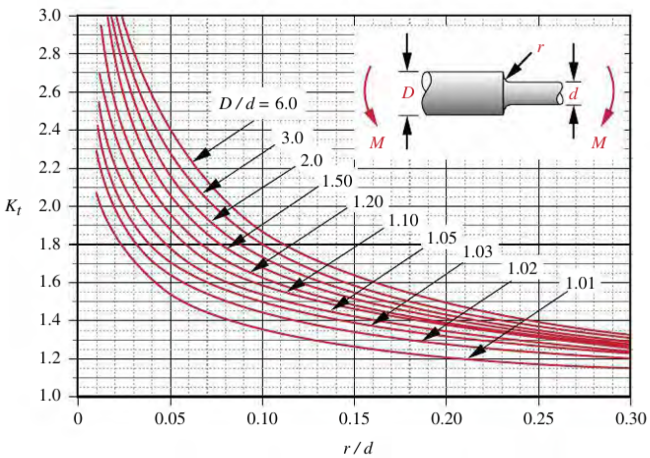
10 Appendix A

Some of the data for the fatigue-stress-life analysis has been extracted from one or more of the following graphs [15]:

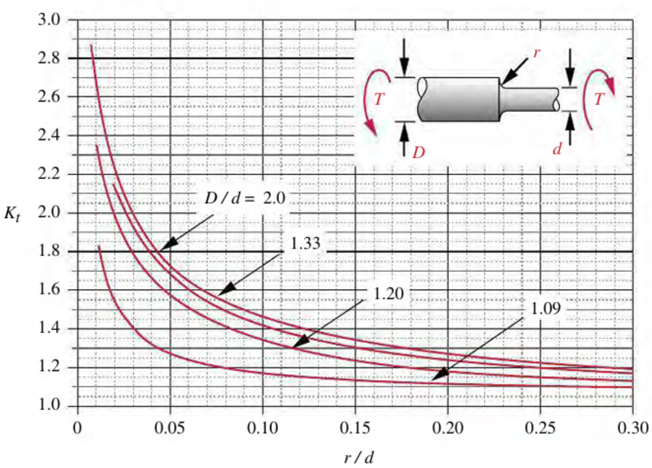


$K_t \cong A\left(\frac{r}{d}\right)^b$

where:

D/d	A	b
6.00	0.878 68	–0.332 43
3.00	0.893 34	–0.308 60
2.00	0.908 79	–0.285 98
1.50	0.938 36	–0.257 59
1.20	0.970 98	–0.217 96
1.10	0.951 20	–0.237 57
1.07	0.975 27	–0.209 58
1.05	0.981 37	–0.196 53
1.03	0.980 61	–0.183 81
1.02	0.960 48	–0.177 11
1.01	0.919 38	–0.170 32

Figure 10.1 Kt of shaft in bending [15]



$K_t \cong A\left(\frac{r}{d}\right)^b$

where:

D/d	A	b
2.00	0.863 31	–0.238 65
1.33	0.848 97	–0.231 61
1.20	0.834 25	–0.216 49
1.09	0.903 37	–0.126 92

Figure 10.2 Kt of beam in torsion [15]

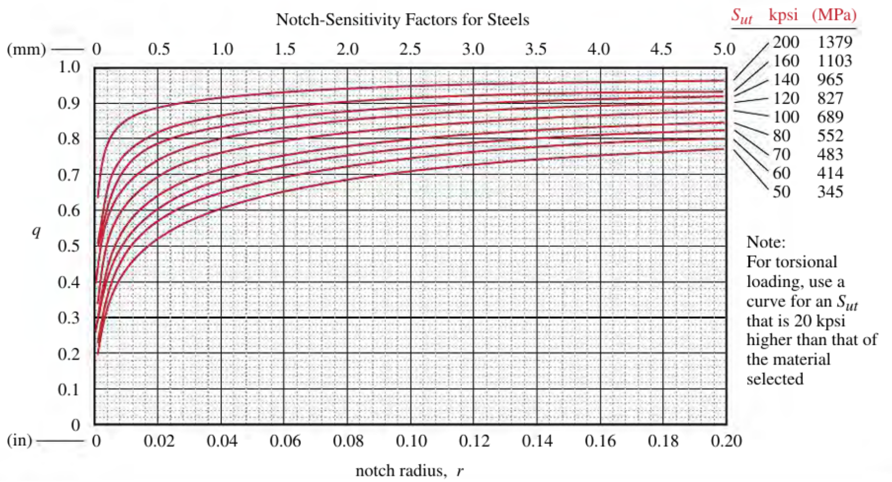


Figure 10.3 Notch sensitivity factor for steels [15]

Table 12-6

Suggested Gear Quality Numbers for Various Applications

Application	Q_v
Cement mixer	3–5
Cement kiln	5–6
Steel mill drives	5–6
Cranes	5–7
Punch press	5–7
Conveyor	5–7
Packaging machinery	6–8
Power drill	7–9
Washing machine	8–10
Printing press	9–11
Automotive transmission	10–11
Marine transmission	10–12
Aircraft engine drive	10–13
Gyroscope	12–14

Figure 10.4 Gear Quality for Various Applications [15]

Table 12-16
Load Distribution Factors K_m

Face Width in (mm)	K_m
<2 (50)	1.6
6 (150)	1.7
9 (250)	1.8
≥20 (500)	2.0

Figure 10.5 Load distribution factors [15]

Table 12-9 AGMA Bending Geometry Factor J for 20°, Full-Depth Teeth with HPSTC Loading

Gear teeth	Pinion teeth: 12		14		17		21		26		35		55		135	
	P	G	P	G	P	G	P	G	P	G	P	G	P	G	P	G
12	U	U														
14	U	U	U	U												
17	U	U	U	U	U	U										
21	U	U	U	U	U	U	0.33	0.33								
26	U	U	U	U	U	U	0.33	0.35	0.35	0.35						
35	U	U	U	U	U	U	0.34	0.37	0.36	0.38	0.39	0.39				
55	U	U	U	U	U	U	0.34	0.40	0.37	0.41	0.40	0.42	0.43	0.43		
135	U	U	U	U	U	U	0.35	0.43	0.38	0.44	0.41	0.45	0.45	0.47	0.49	0.49

Figure 10.6 AGMA Bending Geometry Factor J [15]

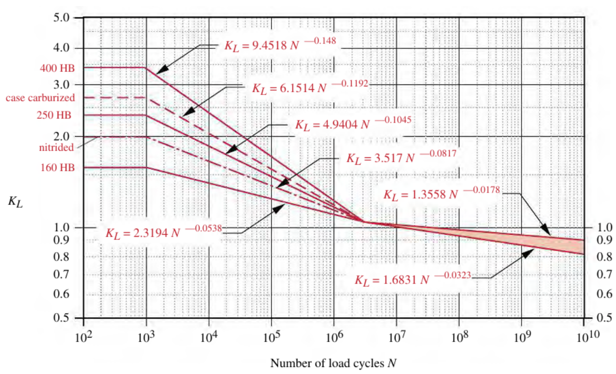


Figure 10.7 KL factors [15]

Reliability Factor K_R

Reliability %	K_R
90	0.85
99	1.00
99.9	1.25
99.99	1.50

Figure 10.8 Reliability factors [15]

Table 12-20 Bending-Fatigue Strengths S_{fb}' for a Selection of Gear Materials*†

Material	Class	Material Designation	Heat Treatment	Minimum Surface Hardness	Bending-Fatigue Strength psi x 10^3	Bending-Fatigue Strength MPa
Steel	A1—A5		Through hardened	≤180 HB	25—33	170—230
			Through hardened	240 HB	31—41	210—280
			Through hardened	300 HB	36—47	250—325
			Through hardened	360 HB	40—52	280—360
			Through hardened	400 HB	42—56	290—390
			Flame or induction hardened	Type A pattern 50—54 HRC	45—55	310—380
			Flame or induction hardened	Type B pattern	22	150
			Carburized and case hardened	55—64 HRC	55—75	380—520
		AISI 4140	Nitrided	84.6 HR15N	34—40	230—310
		AISI 4340	Nitrided	83.5 HR15N	36—47	250—325
		Nitralloy 135M	Nitrided	90.0 HR15N	38—48	260—330
		Nitralloy N	Nitrided	90.0 HR15N	40—50	280—345
		2.5% Chrome	Nitrided	87.5—90.0 15N	55—65	380—450
Cast iron	20	Class 20	As cast		5	34
	30	Class 30	As cast	175 HB	8.5	59
	40	Class 40	As cast	200 HB	13	90
Nodular (ductile) iron	A-7-a	60-40-18	Annealed	140 HB	22—33	152—228
	A-7-c	80-55-06	Quenched and tempered	179 HB	22—33	152—228
	A-7-d	100-70-03	Quenched and tempered	229 HB	27—40	186—276
	A-7-e	120-90-02	Quenched and tempered	269 HB	31—44	213—303
Malleable iron (pearlitic)	A-8-c	45007		165 HB	10	70
	A-8-e	50005		180 HB	13	90
	A-8-f	53007		195 HB	16	110
	A-8-i	80002		240 HB	21	145
Bronze	Bronze 2	AGMA 2C	Sand cast	40 ksi min tensile strength	5.7	40
	Al/Br 3	ASTM B-148 alloy 954	Heat treated	90 ksi min tensile strength	23.6	160

Figure 10.9 Bending fatigue strengths [15]

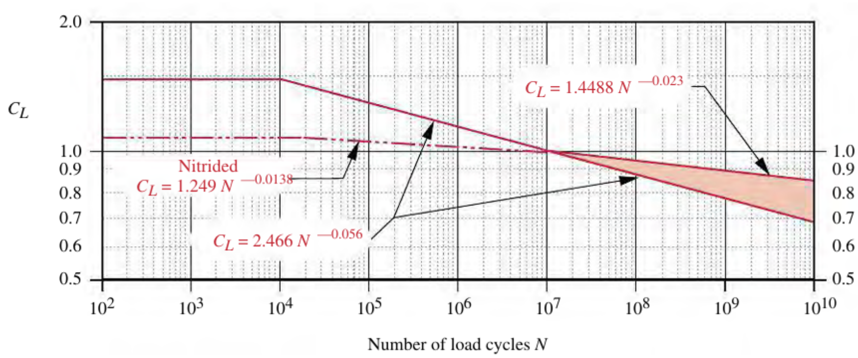


Figure 10.10 CL factors [15]

Table 12-21 Surface-Fatigue Strengths S_{fc}' for a Selection of Gear Materials*†

Material	Class	Material Designation	Heat Treatment	Minimum Surface Hardness	Surface-Fatigue Strength psi x 10^3	Surface-Fatigue Strength MPa
Steel	A1–A5		Through hardened	≤ 180 HB	85–95	590–660
			Through hardened	240 HB	105–115	720–790
			Through hardened	300 HB	120–135	830–930
			Through hardened	360 HB	145–160	1000–1100
			Through hardened	400 HB	155–170	1100–1200
			Flame or induction hardened	50 HRC	170–190	1200–1300
			Flame or induction hardened	54 HRC	175–195	1200–1300
			Carburized and case hardened	55–64 HRC	180–225	1250–1300
		AISI 4140	Nitrided	84.6 HR15N†	155–180	1100–1250
		AISI 4340	Nitrided	83.5 HR15N	150–175	1050–1200
		Nitralloy 135M	Nitrided	90.0 HR15N	170–195	1170–1350
		Nitralloy N	Nitrided	90.0 HR15N	195–205	1340–1410
		2.5% Chrome	Nitrided	87.5 HR15N	155–172	1100–1200
		2.5% Chrome	Nitrided	90.0 HR15N	192–216	1300–1500
Cast iron	20	Class 20	As cast		50–60	340–410
	30	Class 30	As cast	175 HB	65–75	450–520
	40	Class 40	As cast	200 HB	75–85	520–590
Nodular (ductile) iron	A-7-a	60-40-18	Annealed	140 HB	77–92	530–630
	A-7-c	80-55-06	Quenched and tempered	180 HB	77–92	530–630
	A-7-d	100-70-03	Quenched and tempered	230 HB	92–112	630–770
	A-7-e	120-90-02	Quenched and tempered	230 HB	103–126	710–870
Malleable iron (pearlitic)	A-8-c	45007		165 HB	72	500
	A-8-e	50005		180 HB	78	540
	A-8-f	53007		195 HB	83	570
	A-8-i	80002		240 HB	94	650
Bronze	Bronze 2	AGMA 2C	Sand cast	40 ksi min tensile strength	30	450
	Al/Br 3	ASTM B-148 78 alloy 954	Heat–treated	90 ksi min tensile strength	65	450

Figure 10.11 Surface fatigue strengths [15]

Bearing Type			In Relation to the Load the Inner Ring is		Single Row Bearings 1)		Double Row Bearings 2)				e
			Rotat-ing	Station-ary	$\frac{F_a}{VF_r} > e$		$\frac{F_a}{VF_r} \leq e$		$\frac{F_a}{VF_r} > e$		
			V	V	X	Y	X	Y	X	Y	
3) Radial Contact Groove Ball Bearings	4) $\frac{F_a}{C_0}$	5) $\frac{F_a}{i Z D_w^2}$									
	0.014	25				2.30				2.30	0.19
	0.028	50				1.99				1.99	0.22
	0.056	100				1.71				1.71	0.26
	0.084	150				1.55				1.55	0.28
	0.11	200	1	1.2	0.56	1.45	1	0	0.56	1.45	0.30
	0.17	300				1.31				1.31	0.34
	0.28	500				1.15				1.15	0.38
	0.42	750				1.04				1.04	0.42
	0.56	1000				1.00				1.00	0.44
20°					0.43	1.00		1.09	0.70	1.63	0.57
25°					0.41	0.87		0.92	0.67	1.44	0.68
30°			1	1.2	0.39	0.76	1	0.78	0.63	1.24	0.80
35°					0.37	0.66		0.66	0.60	1.07	0.95
40°					0.35	0.57		0.55	0.57	0.93	1.14
Self-Aligning Ball Bearings			1	1	0.40	$0.4 \cot \alpha$	1	$0.42 \cot \alpha$	0.65	$0.65 \cot \alpha$	$1.5 \tan \alpha$
Self-Aligning and Tapered Roller Bearings			1	1.2	0.40	$0.4 \cot \alpha$	1	$0.45 \cot \alpha$	0.67	$0.67 \cot \alpha$	$1.5 \tan \alpha$

1) For single row bearings, when $\frac{F_a}{VF_r} \leq e$ use $X = 1$ and $Y = 0$.

For two single row angular contact ball or roller bearings mounted "face-to-face" or "back-to-back" the values of X and Y which apply to double row bearings. For two or more single row bearings mounted "in tandem" use the values of X and Y which apply to single row bearings.

2) Double row bearings are presumed to be symmetrical.

3) Permissible maximum value of $\frac{F_a}{C_0}$ depends on the bearing design.

4) C_0 is the basic static load rating.

5) Units are pounds and inches.

Values of X, Y and e for a load or contact angle other than shown in the table are obtained by linear interpolation.

Figure 10.12 Factors V, X and Y for Radial Bearings [15]

Table 11-5

Reliability Factors *R* for a Weibull Distribution Corresponding to the Probability of Failure *P*

P%	R%	K_R
50	50	5.0
10	90	1.0
5	95	0.62
4	96	0.53
3	97	0.44
2	98	0.33
1	99	0.21

Figure 10.13 Reliability Factors for Bearings [15]

Table 15-7 Metric Specifications and Strengths for Steel Bolts

Class Number	Size Range Outside Diameter (mm)	Minimum Proof Strength (MPa)	Minimum Yield Strength (MPa)	Minimum Tensile Strength (MPa)	Material
4.6	M5–M36	225	240	400	low or medium carbon
4.8	M1.6–M16	310	340	420	low or medium carbon
5.8	M5–M24	380	420	520	low or medium carbon
8.8	M3–M36	600	660	830	medium carbon, Q&T
9.8	M1.6–M16	650	720	900	medium carbon, Q&T
10.9	M5–M36	830	940	1040	low-carbon martensite, Q&T
12.9	M1.6–M36	970	1100	1220	alloy, quenched & tempered

Figure 10.14 Metric Specification and Strengths for Steel Bolts

Table 15-2 Principal Dimensions of ISO Metric Standard Screw Threads
Data Calculated from Equations 15.1—See Reference 4 for More Information

	Coarse Threads			Fine Threads		
Major Diameter d (mm)	Pitch p mm	Minor Diameter d_r (mm)	Tensile Stress Area A_t (mm^2)	Pitch p mm	Minor Diameter d_r (mm)	Tensile Stress Area A_t (mm^2)
3.0	0.50	2.39	5.03			
3.5	0.60	2.76	6.78			
4.0	0.70	3.14	8.78			
5.0	0.80	4.02	14.18			
6.0	1.00	4.77	20.12			
7.0	1.00	5.77	28.86			
8.0	1.25	6.47	36.61	1.00	6.77	39.17
10.0	1.50	8.16	57.99	1.25	8.47	61.20
12.0	1.75	9.85	84.27	1.25	10.47	92.07
14.0	2.00	11.55	115.44	1.50	12.16	124.55
16.0	2.00	13.55	156.67	1.50	14.16	167.25
18.0	2.50	14.93	192.47	1.50	16.16	216.23
20.0	2.50	16.93	244.79	1.50	18.16	271.50
22.0	2.50	18.93	303.40	1.50	20.16	333.06
24.0	3.00	20.32	352.50	2.00	21.55	384.42
27.0	3.00	23.32	459.41	2.00	24.55	495.74
30.0	3.50	25.71	560.59	2.00	27.55	621.20
33.0	3.50	28.71	693.55	2.00	30.55	760.80
36.0	4.00	31.09	816.72	3.00	32.32	864.94
39.0	4.00	34.09	975.75	3.00	35.32	1028.39

Figure 10.15 Principal Dimensions of ISO Metric Standard Screw Threads

Table 15-8 Parameters for Equation 15.19 [15]

j	p_0	p_1	p_2	p_3
0.10	0.4389	−0.9197	0.8901	−0.3187
0.20	0.6118	−1.1715	1.0875	−0.3806
0.30	0.6932	−1.2426	1.1177	−0.3845
0.40	0.7351	−1.2612	1.1111	−0.3779
0.50	0.7580	−1.2632	1.0979	−0.3708
0.60	0.7709	−1.2600	1.0851	−0.3647
0.70	0.7773	−1.2543	1.0735	−0.3595
0.80	0.7800	−1.2503	1.0672	−0.3571
0.90	0.7797	−1.2458	1.0620	−0.3552
1.00	0.7774	−1.2413	1.0577	−0.3537
1.25	0.7667	−1.2333	1.0548	−0.3535
1.50	0.7518	−1.2264	1.0554	−0.3550
1.75	0.7350	−1.2202	1.0581	−0.3574
2.00	0.7175	−1.2133	1.0604	−0.3596

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Figure 10.16 Parameters for Joint Stiffness Factor

11 Appendix B – Detailed Calculations

11.1 Shear and Moment Calculations for input and output Shafts

All unknown forces assumed positive in calculations, FBD represents resultant directions of forces. All Calculations for the outlined tables were performed as such, where l_n represents the lengths outlined in figures 12.1 through to 12.2.

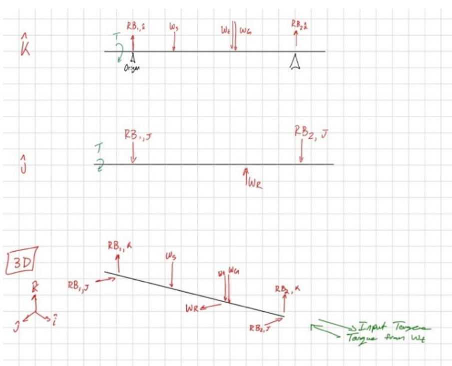

Figure 11.1 FBD of shaft I

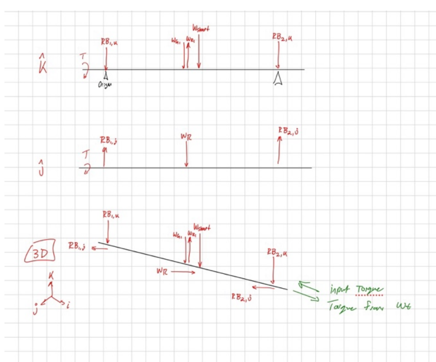

Figure 11.2 FBD of shaft O

$$V_k = R_{B1,k}\langle x-0\rangle^0 + F_{gear,k}\langle x-(l_2+l_3+l_4)\rangle^0 + W_s\langle x-(shaft\ centroid)\rangle^0 + R_{B2,k}\langle x-(l_2+l_3+l_4+l_5+l_6+l_7)\rangle^0$$

$$V_j = R_{B1,j}\langle x-0\rangle^0 + F_{gear,j}\langle x-(l_2+l_3+l_4)\rangle^0 + R_{B2,j}\langle x-(l_2+l_3+l_4+l_5+l_6+l_7)\rangle^0$$

$$M_k = R_{B1,k}\langle x-0\rangle + F_{gear,k}\langle x-(l_2+l_3+l_4)\rangle + W_s\langle x-(shaft\ centroid)\rangle + R_{B2,k}\langle x-(l_2+l_3+l_4+l_5+l_6+l_7)\rangle$$

$$M_j = R_{B1,j}\langle x - 0\rangle + F_{gear,j}\langle x - (l_2 + l_3 + l_4)\rangle + R_{B2,j}\langle x - (l_2 + l_3 + l_4 + l_5 + l_6 + l_7)\rangle$$

$$R_{B1,k} = -R_{B2,k} - F_{gear,k} - W_s$$

$$R_{B1,j} = -R_{B2,j} - F_{gear,j}$$

$$R_{B2,k} = -\frac{F_{gear,k}(l_2 + l_3 + l_4) + W_s(shaft\ centroid)}{(l_2 + l_3 + l_4 + l_5 + l_6 + l_7)}$$

$$R_{B2,j} = -\frac{F_{gear,j}(l_2 + l_3 + l_4)}{(l_2 + l_3 + l_4 + l_5 + l_6 + l_7)}$$

$$F_{gear,k} = W_T + W_G$$

$$F_{gear,j} = W_R$$

$$W_s = -(mass\ of\ shaft)(gravity)\ \ ,\ force\ located\ at\ shaft\ centroid$$

$$W_G = -(mass\ of\ gear)(gravity)$$

11.2 Shear and Moment Calculations for Shafts A and B

All unknown forces assumed positive in calculations, FBD represents the resultant directions of forces. All Calculations for the outlined tables were performed as such, where l_n represents the lengths outlined in figures 12.3 through to 12.4.

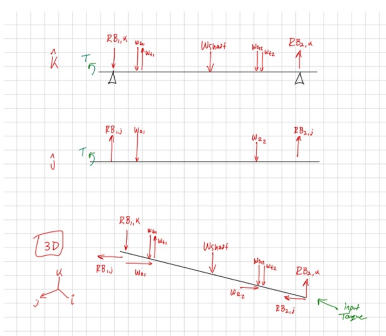

Figure 11.3 FBD of shaft A

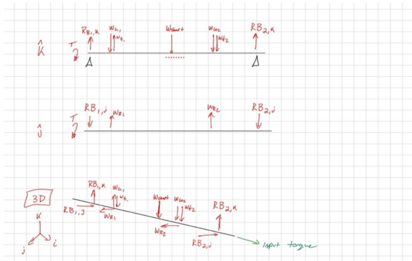

Figure 11.4 FBD of shaft B

$$V_k = R_{B1,k}\langle x - 0\rangle^0 + F_{gear1,k}\langle x - (l_2 + l_3 + l_4)\rangle^0 + F_{gear2,k}\langle x - (l_2 + l_3 + l_4 + l_5 + l_6 + l_7)\rangle^0 + W_s\langle x - (shaft\ centroid)\rangle^0 + R_{B2,k}\langle x - (l_2 + l_3 + l_4 + l_5 + l_6 + l_7 + l_8 + l_9 + l_{10})\rangle^0$$

$$V_j = R_{B1,j}\langle x - 0\rangle^0 + F_{gear1,j}\langle x - (l_2 + l_3 + l_4)\rangle^0 + F_{gear2,j}\langle x - (l_2 + l_3 + l_4 + l_5 + l_6)\rangle^0 + R_{B2,j}\langle x - (l_2 + l_3 + l_4 + l_5 + l_6 + l_7 + l_8 + l_9 + l_{10})\rangle^0$$

$$M_k = R_{B1,k}\langle x - 0\rangle + F_{gear1,k}\langle x - (l_2 + l_3 + l_4)\rangle + F_{gear2,k}\langle x - (l_2 + l_3 + l_4 + l_5 + l_6 + l_7)\rangle + W_s\langle x - (shaft\ centroid)\rangle + R_{B2,k}\langle x - (l_2 + l_3 + l_4 + l_5 + l_6 + l_7 + l_8 + l_9 + l_{10})\rangle$$

$$M_j = R_{B1,j}\langle x - 0\rangle + F_{gear1,j}\langle x - (l_2 + l_3 + l_4)\rangle + F_{gear2,j}\langle x - (l_2 + l_3 + l_4 + l_5 + l_6 + l_7)\rangle + R_{B2,j}\langle x - (l_2 + l_3 + l_4 + l_5 + l_6 + l_7 + l_8 + l_9 + l_{10})\rangle$$

$$R_{B1,k} = -R_{B2,k} - F_{gear1,k} - F_{gear2,k} - W_s$$

$$R_{B1,j} = -R_{B2,j} - F_{gear1,j} - F_{gear2,j}$$

$$R_{B2,k} = -\frac{F_{gear1,k}(l_2 + l_3 + l_4) + F_{gear2,k}(l_2 + l_3 + l_4 + l_5 + l_6 + l_7) + W_s(shaft\ centroid)}{(l_2 + l_3 + l_4 + l_5 + l_6 + l_7 + l_8 + l_9 + l_{10})}$$

$$R_{B2,j} = -\frac{F_{gear1,j}(l_2 + l_3 + l_4) + F_{gear2,j}(l_2 + l_3 + l_4 + l_5 + l_6 + l_7)}{(l_2 + l_3 + l_4 + l_5 + l_6 + l_7 + l_8 + l_9 + l_{10})}$$

$$F_{gear1,k} = W_{T1} + F_{G1}$$

$$F_{gear1,j} = W_{R1}$$

$$F_{gear2,k} = W_{T2} + F_{G2}$$

$$F_{gear2,j} = W_{R2}$$

$$W_s = -(mass\ of\ shaft)(gravity)\ \ ,\ force\ located\ at\ shaft\ centroid$$

$$F_{G1} = -(mass\ of\ gear\ 1\ )(gravity)$$

$$F_{G2} = -(mass\ of\ gear\ 2\ )(gravity)$$

11.3 Gear Bending and Surface Safety Factor Calculations

$$N_b = \frac{S_{fb}}{\sigma_b}$$

$$N_c = (\frac{S_{fc}}{\sigma_c})^2$$

11.3.1 Gear Material Specifications

Gear Material: AISI 9310, Carburized and Case Hardened to 60-62 HRC [14], This material is considered a class A1 steel, and can be Carburized and Case Hardened.

$$posson's\ ratio, v = 0.285$$

$$Modulus\ of\ elasticity, E = 200\ GPa$$

11.3.2 Bending Safety Factor Calculations

$$S_{fb} = \frac{K_L}{K_T K_R} S_{fb'}$$

$$K_L = 1.3558N^{-0.0178}\ \rightarrow assumed\ commercial\ application$$

$$N = (25\ years)(365.25\ days)(24\ hours)(60\ minutes)(w\frac{30}{\pi})$$

$$K_T = 1\ \rightarrow oil\ temps\ \leq 250°\text{F}$$

$$K_R = 1\ \rightarrow 99\%\ reliability$$

$$S_{fb'} = 450Mpa\ \rightarrow A4\ carburized\ and\ case\ hardened\ steel$$

11.3.3 Surface Safety Factor Calculations

$$S_{fc} = \frac{C_L C_H}{C_T C_R} S_{fc'}$$

$$C_L = 1.4488N^{-0.023} \rightarrow commercial\ application$$

$$N = (25\ years)(365.25\ days)(24\ hours)(60\ minutes)(w\frac{30}{\pi})$$

$$C_H = 1 \rightarrow HB_p = HB_g$$

$$C_T = K_T$$

$$C_R = K_R$$

$$S_{fc'} = 1250\ MPa \rightarrow A4\ carburized\ and\ case\ hardened\ steel$$

11.3.4 Bending Stress Calculations

$$\sigma_b = \frac{W_T}{FmJ}\frac{K_a K_m}{K_v}K_s K_B K_I$$

$$k_v = \left(\frac{A}{A + \sqrt{200v_t}}\right)^B, Q_v = 11$$

$$A = 50 + 56(1 - B)$$

$$B = \frac{(12 - Q_v)^{2/3}}{4}$$

$$v_t = wr$$

$$J = figure\ 11.5$$

$$K_m = figure\ 11.6$$

$$F = \frac{12}{p_d} \rightarrow nominal\ value\ will\ adjust\ to\ alter\ safety\ factor$$

$$K_a = 1 \rightarrow Uniform\ Appplication$$

$$K_s = 1.25 \rightarrow Teeth\ are\ large$$

$$K_B = 1 \rightarrow Solid\ disk$$

$$K_I = 1 \rightarrow No\ idler\ gears$$

11.3.5 Surface Stress Calculations

$$\sigma_c = C_p\sqrt{\frac{W_T}{FId}\frac{C_a C_m}{C_v}C_s C_f}$$

$$I = \frac{\cos\phi}{\left(\frac{1}{\rho_p} + \frac{1}{\rho_g}\right) d_p}$$

$$\rho_p = \sqrt{\left(r_p + \frac{1 + x_p}{p_d}\right)^2 - \left(r_p \cos\phi\right)^2} - \frac{\pi}{p_d}\cos\phi$$

$$\rho_g = C\sin\phi - \rho_p$$

$$x_p = 0 \rightarrow full\ depth\ teeth$$

$$p_d = \frac{25.4}{m}$$

$$r_p = pitch\ radius$$

$$d_p = 2r_p$$

$$C_p = \sqrt{\frac{1}{2\pi\left(\frac{1 - v^2}{E}\right)}}$$

$$C_F = 1 \rightarrow Conventional\ surface\ finish$$

$$C_s = K_s$$

$$C_a = K_m$$

$$C_v = K_v$$

$$C_m = K_m$$

11.4 Contact Ratio Calculations

$$m_p = \frac{p_d Z}{\pi \cos\phi}$$

$$Z = \sqrt{(r_p + a_p)^2 - (r_p\cos\phi)^2} + \sqrt{(r_g + a_g)^2 - (r_g\cos\phi)^2} - C\sin\phi$$

$$C = r_p + r_g$$

11.5 Fastener Calculations

Fastener calculations were done using a M36x4 class 12.9 steel bolt as specified in section 7.9 above.

$$S_{ult} = 1220\,MPa \quad , \quad see\ figure\ 11.14\ in\ Appendix\ A$$

$$S_y = 1100\,MPa \quad , \quad see\ figure\ 11.14\ in\ Appendix\ A$$

$$S_b = 970\,MPa \quad , \quad see\ figure\ 11.14\ in\ Appendix\ A$$

$$d = 36mm$$

$$A_t = 816.72mm^2 \quad , \quad see\ figure\ 11.15\ in\ appendix\ A$$

11.5.1 Safety Factors

$$Nf = \frac{S_e(S_{ut} - \sigma_i)}{S_e(\sigma_m - \sigma_i) + S_{ut}\sigma_a}$$

$$N_{Yeilding} = \frac{S_y}{\sigma_b}$$

$$N_{seperation} = \frac{F_i}{P(1 - C)}$$

11.5.2 Stresses

$$\sigma_a = k_f \frac{F_a}{A_t}$$

$$\sigma_m = k_{fm} \frac{F_m}{A_t}$$

$$\sigma_i = k_{fm} \frac{F_i}{A_t}$$

$$\sigma_b = \frac{F_b}{A_t}$$

11.5.3 Internal Forces

$$F_a = \frac{F_b - F_i}{2}$$

$$F_m = \frac{F_b + F_i}{2}$$

$$F_i = f_b S_b A_t$$

$$f_b = 0.75$$

$$F_b = F_b + CP$$

11.5.4 External Load

$$Reaction\ at\ Bearing\ 1 = \sqrt{R_y^2 + R_z^2}$$

$$Reaction\ at\ Bearing\ 2 = \sqrt{R_y^2 + R_z^2}$$

$$Max\ External\ Load,\ P\ \text{max} = \frac{Reaction\ at\ Bearing\ 1 + Reaction\ at\ Bearing\ 2}{Number\ of\ Bolts}$$

11.5.5 Joint Stiffness Factor

Calculations done based on the two plates of the same material

$$C = p_3 r^3 + p_2 r^2 + p_1 r + p_0$$

$$j = \frac{d}{l}$$

$$l = 90mm$$

$$r = \frac{E_{material}}{E_{bolt}} = 1$$

$$p_0, p_1, p_2, p_3 = see\ figure\ 11.16\ in\ appendix\ A$$

11.5.6 Correction Factors

$$k_f = 5.7 + 0.02682d$$

$$k_{fm} \cong 1 \rightarrow Preloaded$$

11.5.7 Ultimate Strength

$$S_e = C_{LOAD} C_{SIZE} C_{SURF} C_{TEMP} C_{RELI} S_{e'}$$

$$S_{e'} \approx 0.5 S_{UT} \approx 610MPa$$

$$C_{LOAD} = 0.7 \rightarrow Axial\ Loading$$

$$C_{SIZE} = 1 \rightarrow Axial\ Loading$$

$$C_{SURF} \cong A(S_{UT})^b \cong 4.51(S_{ut})^{-0.265} = 0.69$$

$$C_{TEMP} = 1.0$$

$$C_{RELI} = 0.814 \rightarrow 99\%$$

11.6 Bearing Calculations for Table

$$P = X.Fr + Y.Fa$$

$$X = 1 \rightarrow Deep\ Groove\ Ball\ Bearings$$

$$Y = 0.56 \rightarrow Deep\ Groove\ Ball\ Bearings$$

$$L_{10} = \left(\frac{Cr}{P}\right)^3 \cdot 0.21 \rightarrow Used\ a\ Reliability\ Factor\ of\ 99\%$$

$$L_{10,hours} = \left(\frac{L_{10}.\ 10^6}{N.60}\right)$$

12 Appendix C – Bearing Nuts, Lock washers, Collars and Seals

The bearing nuts and their respective lock washers were all chosen from McMaster-Carr other than the output shaft which was found on SKF. Specifications can be seen in table 13.1 below.

Table 12.1 Bearing Nut and Lock Washer Specifications

Shaft	Bearing Nut PN	Lock Washer PN
I	McMaster 3554N42	McMaster 90391A142
A	McMaster 3554N31	McMaster 90391A131
B	McMaster 3554N37	McMaster 90391A137
O	SKF HM 3168	Custom

All collars used are standard, two-piece clamp-down collars. As we could not find any collars to fit the large shaft diameters, they were made custom for all shafts. See McMaster-Carr 8386K651 for design inspiration of all collars

The seals for the input and output shaft are the SKF 130X230X14 HMSA10 V and SKF 320X360X20 HMSA10 V respectively.